Algorithms for Data Science: Itemsets

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Data Mining

Next two lectures: the analysis of itemsets

- Similar itemsets
- Frequent itemsets
Similar Itemsets

Collaborative Filtering

- Items: customers
- Itemsets: customers that bought a specific book
- Similar itemsets: books purchased by same customers
Similar Itemsets

Similar Documents

- Items: words
- Itemsets: documents
- Similar itemsets: documents using many of the same words
Similar Itemsets

Similar News Articles

- Items: words
- Itemsets: news articles
- Similar itemsets: news articles using many of the same words
Online Purchasing

- Items: books
- Itemsets: orders (sets of books)
- Frequent itemsets: sets of books that are purchased together frequently
Similar Itemsets

- Describing set similarity (Jaccard Similarity)
- Representing documents as sets (Shingling)
- Similarity-preserving set summaries (Minhash)
- Search for similar itemsets using Locality-Sensitive Hashing (LSH)
Jaccard Similarity

The *Jaccard Similarity* of sets $S$ and $T$ is

$$\frac{S \cap T}{S \cup T}$$
Exercises

• Compute the Jaccard similarities of each pair of sets: \{1, 2, 3, 4\}, \{2, 3, 5, 7\}, \{2, 4, 6\}
Documents (Shingles)

- Set all words to lowercase, remove all whitespace and punctuation

"Hurricane Irma, they confirmed landfall" -> "hurricaneirmatheyconfirmedlandfall"
Documents (Shingles)

- Set all words to lowercase, remove all whitespace and punctuation

"Hurricane Irma, they confirmed landfall" -> "hurricaneirmatheyconfirmedlandfall"

- For some parameter $k$, represent document as the set of $k$-long subsequences of document

For $k=3$

{$hur,urr,rri,...,eir,irm,rma,...,fir,irm,rme,...$}
Documents (Shingles)

- Choosing $k$: choose large enough that probability of any given shingle appearing in any given document is low. Depends on collection.
- Hashing: hash $k$-shingles instead of using them directly in algorithms that follow
- Using words, effective for similarity (more meaning) but sparser, set of possible shingles is huge
Min-Hash

Clever idea: let's summarize item(sets) (reduce data size!) but make it easy to find similar item(sets).
Min-Hash

Characteristic Matrix

<table>
<thead>
<tr>
<th>Element</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>b</td>
<td>0</td>
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<td>1</td>
<td>0</td>
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<td>c</td>
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<td>d</td>
<td>1</td>
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<td>e</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Min-Hash

- Permute the rows of the characteristic matrix
- Min-Hash value of set: first non-zero row in corresponding column
Min-Hash

Permuted characteristic Matrix

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$\text{min\_hash}(S_1) = a$, $\text{min\_hash}(S_2) = c$
Min-Hash

Property: $Pr(\text{min\_hash}(S_i) = \text{min\_hash}(S_j)) = JS(S_i, S_j)$

Pf: on the board
Min-Hash Signatures

- Choose $n$ permutations of rows, and set $h_i(S_j)$ as the Min-Hash given by permutation $i$ of set $j$

- Represent set $j$ by the signature vector of Min-hashes $[h_1(S_j), \ldots, h_n(S_j)]$

- Collect signature vectors into a signature matrix
Min-Hash Signatures in Practice

Instead of row permutations, use hash functions $h_i$ over row indices

Let $SIG(i, c)$ be the $i$th hash of $c$th element

Initialize: set $SIG(i, c) = \infty$ for all $i$ and $c$

Row $r$:

- Compute $h_i(r)$ for all $i$
- For each column $c$:
  - If $c$ has a 0 in row $r$, do nothing
  - If $c$ has a 1 in row $r$, then for each $i = 1, \ldots, n$:
    - set $SIG(i, c)$ to $\min(SIG(i, c), h_i(r))$
Exercise

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<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
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</table>

$h_1(x) = 2x + 1 \mod 6 \quad h_2(x) = 3x + 2 \mod 6 \quad h_3(x) = 5x + 2 \mod 6$
JS and Minhashing

Estimate $JS(S_i, S_j)$ as the proportion of rows (hashes) of the signature matrix that are equal for columns $S_i$ and $S_j$. 
Exercise

Prove that if the JS of two sets is 0, then Min-Hash always gives the right answer.
Locality-Sensitive Hashing

Minhash gives a compressed representation of item(sets) that retains similarity
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But to find all pairs of similar item(sets) can still take a lot of time
Locality-Sensitive Hashing

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But to find all pairs of similar item(sets) can still take a lot of time

LSH gives us a way of only comparing likely similar pairs. Conversely, ignore pairs that are unlikely similar
LSH for Minhash

- Divide signature matrix into $b$ bands, each with $r$ rows
- For each column (itemset) and band, hash it's $r$ entries according to some hash function
- Use same hash function in each of the bands, but use different hash arrays
LSH for Minhash

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Itemsets with similar signatures will hash to the same bucket with some likelihood (candidate similar pair)
LSH for Minhash

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Itemsets with similar signatures will hash to the same bucket with some likelihood (candidate similar pair)

Itemsets without matching signatures will not
Analysis of LSH

Let $JS(S, T) = s$

- Probability signatures agree in all rows of one band: $s^r$
- Probability do not agree in at least one row of a band: $1 - s^r$
- Probability that signatures do not agree in all rows of any of the bands: $(1 - s^r)^b$
- Probability that signatures agree in all the rows of at least one band (hash to the same bucket at least once): $1 - (1 - s^r)^b$. 
Analysis of LSH
Final algorithm for similar document search

Part I: Shingles

- Pick a value of $k$, construct $k$-shingles for each document (optionally hashing $k$-shingles)
- Sort documents by document-shingle pairs by shingle
Final algorithm for similar document search

Part II: Minhash

- Pick a length $n$ for minhash signatures
- Compute minhash signatures for all documents
Final algorithm for similar document search

Part III: LSH

- Choose threshold $t$ for how similar documents have to be to consider as a similar pair
- Choose number of bands $b$ and number of rows $r$ such that $br = n$ and threshold $t$ is approximately $(1/b)^{1/r}$
- Construct candidate pairs using LSH
Final algorithm for similar document search

Part IV: Confirm similar pairs

- For each candidate pair, confirm that their signatures match in at least $t$ fraction of rows
- Optionally, verify similarity in shingled documents
Frequent Itemsets

Find items that occur frequently together in sets

Examples:

- items frequently bought together in the same transaction
- words that appear frequently together in the same document
Market-Basket Model

Items: objects we are modeling
Baskets: sets of items (transactions)

Frequent itemsets: items that co-occur frequently in baskets
Frequent Itemsets

support($I$) the number of baskets in which itemset $I$ appears

Frequent itemsets: Itemsets $I$ with $\text{support}(I) \geq s$ (for some threshold $s$)
Example

(1) \{\text{Cat, and, dog, bites}\}
(2) \{\text{Yahoo, news, claims, a, cat, mated, with, a, dog, and, produced, viable, offspring}\}
(3) \{\text{Cat, killer, likely, is, a, big, dog}\}
(4) \{\text{Professional, free, advice, on, dog, training, puppy, training}\}
(5) \{\text{Cat, and, kitten, training, and, behavior}\}
(6) \{\text{Dog, &, Cat, provides, dog, training, in, Eugene, Oregon}\}
(7) \{\text{Shop, for, your, show, dog, grooming, and, pet, supplies}\}

For $s = 3$ what are frequent itemsets of size 1 or 2
Exercise

Suppose there are 100 items, numbered 1 to 100, and also 100 baskets, numbered 1 to 100.

Item $i$ is in basket $B$ iff $i$ divides $B$ with no remainder

- Item 1 is in all baskets, item 2 in the even-numbered baskets
- Basket 24 contains items $\{1, 2, 3, 4, 6, 8, 12, 24\}$

a) If support threshold is 5, which items are frequent? b) Which pairs are frequent?
Association Rules

Rules of the form $I \rightarrow j$: if itemset $I$ is in basket, then item $j$ is likely in basket as well

*rule confidence:*

$$\text{confidence}(I \rightarrow j) = \frac{\text{support}(I \cup \{j\})}{\text{support}(I)}$$

*rule interest:*

$$\text{confidence}(I \rightarrow j) - |\{B \in \text{Baskets s.t. } j \in B\}|$$
Association Rules

Note: once we have frequent itemsets, we can get association rules with high confidence easily
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Suppose we find all frequent itemsets over some support threshold $s$

Let itemset $J$ with $n$ items be one of those itemsets ($\text{support}(J) \geq s$), then

1. there are only $n$ candidate Association Rules ($J - \{j\}) \rightarrow j$
2. $\text{support}(J) \geq s$ implies $\text{support}(J - \{j\}) \geq s$ so we must have already calculated the latter
3. We can quickly compute the confidence and interest of each rule
Itemset monotonicity

*If $I$ is a frequent itemset, then every subset of $I$ is a frequent itemset*

Why?
The A-priori algorithm

Suppose we are given a set of baskets over \( n \) items

First pass

Count the number of occurrences of each item (array of \( n \) values)

After first pass

Identify frequent singletons (above support threshold)
The A-priori algorithm

Second pass

Count the number of occurrences of pairs of frequent items

- For each basket:
  - Check which of its items are frequent (first pass)
  - For each pair of items increase occurrence count

After the second pass

Identify frequent pairs (above support threshold)
The A-priori algorithm

Third pass

Count the number of occurrences of frequent pairs + a frequent item

- For each basket:
  - Check which item pairs and singletons that are frequent (first and second pass)
  - For each combination of pair and singleton, increase occurrence count

After the third pass

Identify frequent triples (above support threshold)
The A-priori algorithm

And so on until no more frequent sets are identified

Notes:

- The data structure to store pair counts will be important consideration
- The algorithm has a construct-filter structure: at each pass, construct the set of candidate itemsets, filter to those that are frequent
Exercise

Apply A-priori algorithm to previous exercise
Handling large datasets

For large datasets storing occurrences of candidate frequent pairs is problematic

PCY algorithm: hash item pairs and keep count in hash bucket

Define candidate frequent pairs as

- $i$ and $j$ are frequent items
- $\{i, j\}$ hashes to a frequent bucket (with count > threshold)
Handling large datasets

Identify frequent buckets with a bitmap (little memory)

Only count (and verify) candidate pairs as defined above (expected to be much fewer)
Exercise

Consider baskets over items 1, \ldots, 6

\{1, 2, 3\} \ \{2, 3, 4\} \ \{3, 4, 5\} \ \{4, 5, 6\}
\{1, 3, 5\} \ \{2, 4, 6\} \ \{1, 3, 4\} \ \{2, 4, 5\}
\{3, 5, 6\} \ \{1, 2, 4\} \ \{2, 3, 5\} \ \{3, 4, 6\}

- Compute support for each item and each pair of items

- Using hash function \( i \times j \mod 11 \) (hash table with 11 buckets), which pairs hash to the same buckets?
Exercise

- Which buckets are frequent?
- Which pairs are counted in the second pass of PCY algorithm?
Summary

Itemset analysis: applications to collaborative filtering, recommendation engines

Finding Similar Itemsets

- Jaccard similarity: measure of set similarity based on common items
- Minhashing with LSH: effective way of finding similar itemsets with efficient data structures for large datasets
Summary

Finding Frequent Itemsets

- Market-basket data: model of item transactions
- Frequent Itemsets: Sets of items appearing frequently in "baskets"
- Association Rules: $I \rightarrow j$
- Pair-counting Bottleneck: frequent itemset mining memory space taken mostly in keeping counts of pairs of frequent items
- Monotonicity of frequent itemsets
- A-priori Algorithm
- Hashing for large datasets