Algorithms for Data Science: Itemsets

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Data Mining

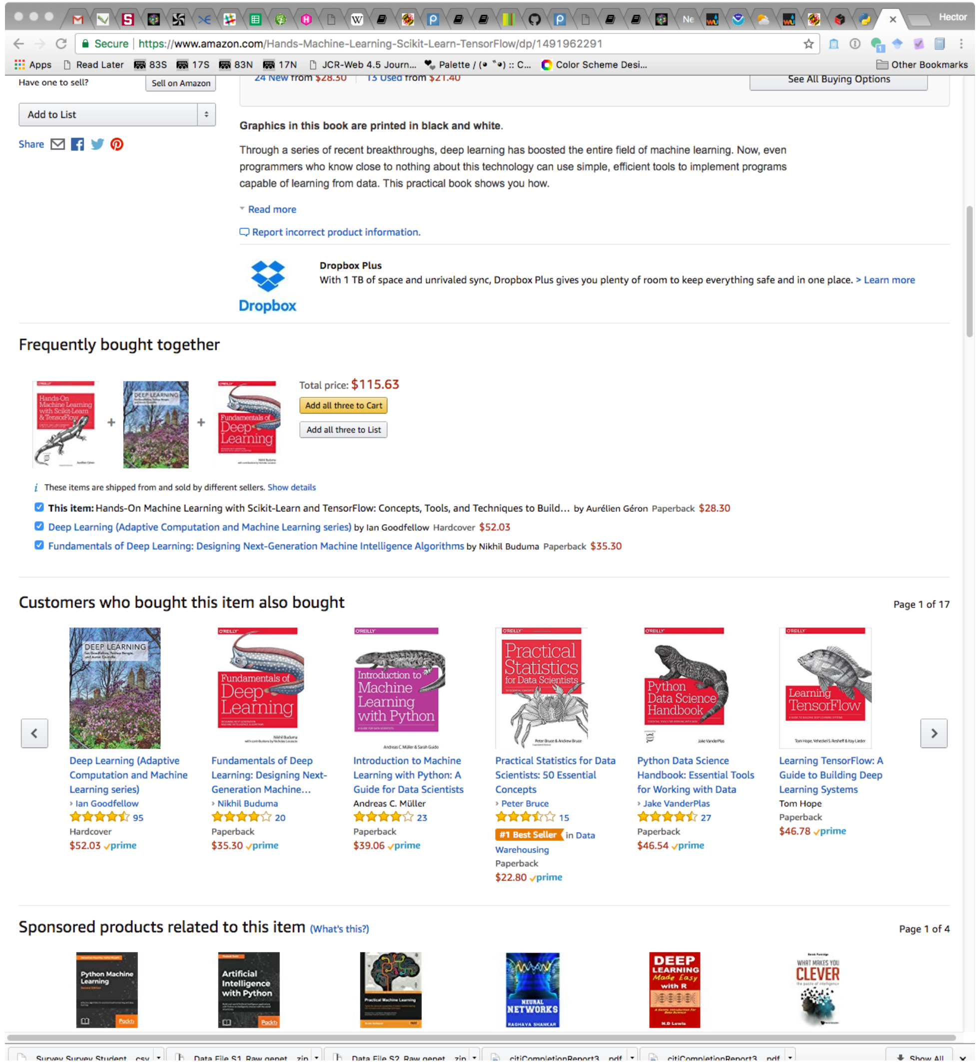
Next two lectures: the analysis of itemsets

- Similar itemsets
- Frequent itemsets

Similar Itemsets

Collaborative Filtering

- Items: customers
- Itemsets: customers that bought a specific book
- Similar itemsets: books purchased by same customers



Similar Itemsets

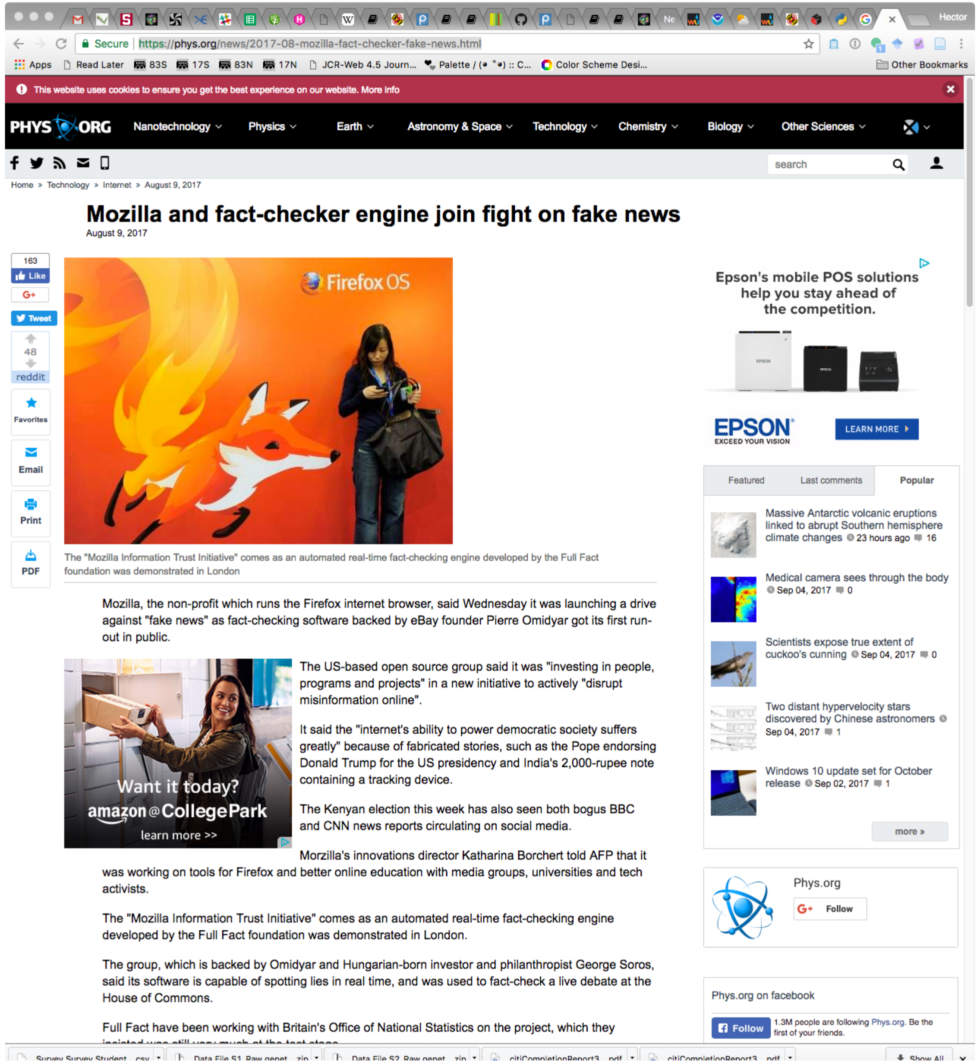
Similar Documents

- Items: words
- Itemsets: documents
- Similar itemsets: documents using many of the same words

Similar Itemsets

Similar News Articles

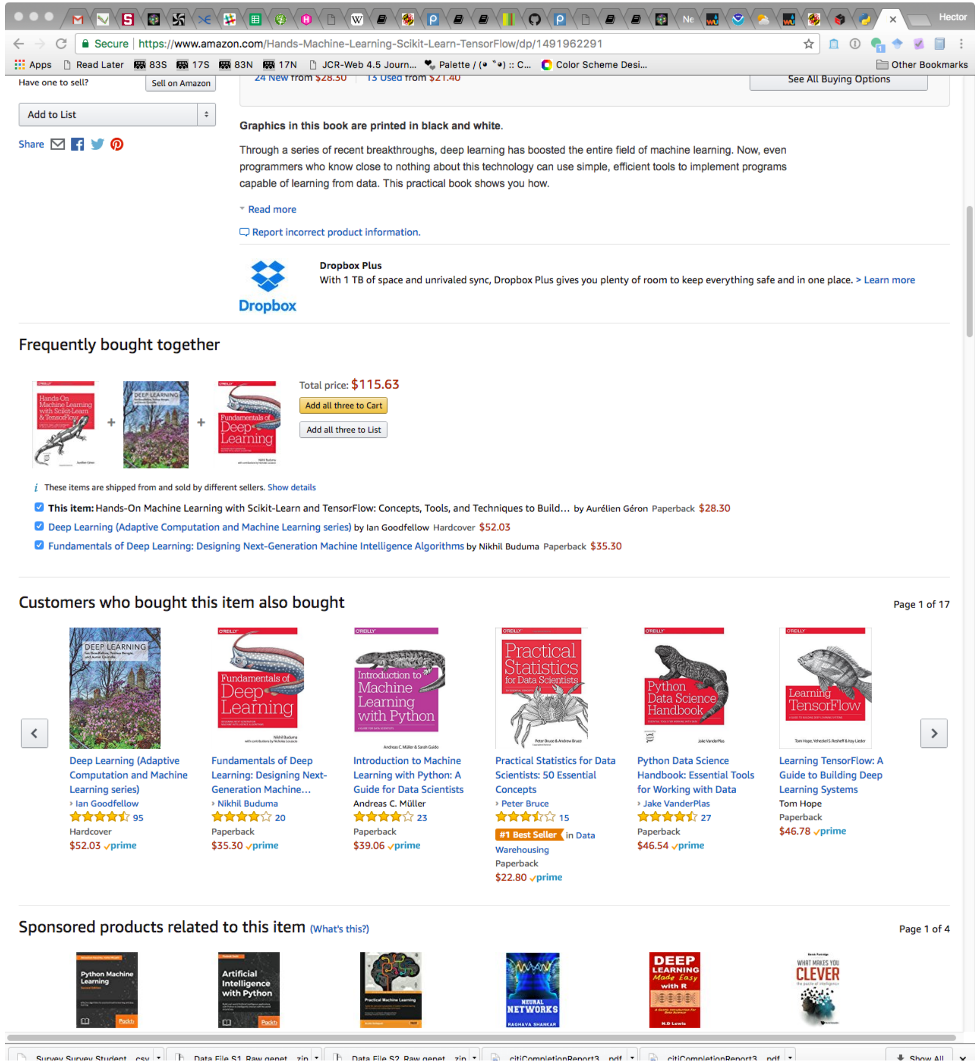
- Items: words
- Itemsets: news articles
- Similar itemsets: news articles usinbg many of the same words



Frequent Itemsets

Online Purchasing

- Items: books
- Itemsets: orders (sets of books)
- Frequent itemsets: sets of books that are purchased together frequently



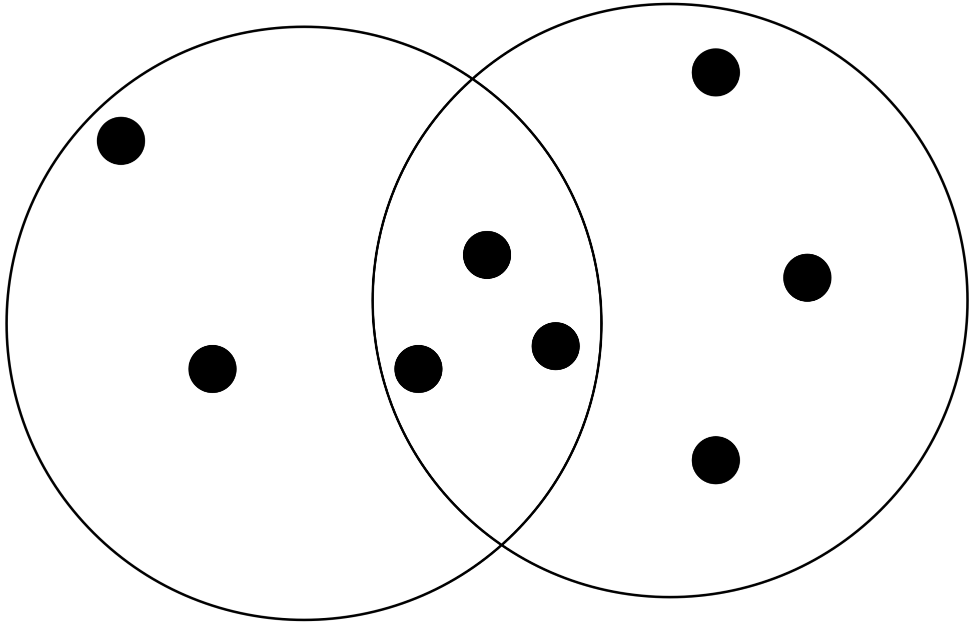
Similar Itemsets

- Describing set similarity (Jaccard Similarity)
- Representing documents as sets (Shingling)
- Similarity-preserving set summaries (Minhash)
- Search for similar itemsets using Locality-Sensitive Hashing (LSH)

Jaccard Similarity

The *Jaccard Similarity* of sets S and T is

$$\frac{S \bigcap T}{S \bigcup T}$$



Exercises

- Compute the Jaccard similarities of each pair of sets: $\{1, 2, 3, 4\}$, $\{2, 3, 5, 7\}$, $\{2, 4, 6\}$

Documents (Shingles)

- Set all words to lowercase, remove all whitespace and punctuation

```
"Hurricane Irma, they confirmed landfall" ->
"hurricaneirmatheyconfirmedlandfall"
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- For some parameter k, represent document as the set of k-long subsequences of document

For k=3

```
{hur,urr,rri,...,eir,irm,rma,...,fir,irm,rme,...}
```

Documents (Shingles)

- Choosing k: choose large enough that probability of any given shingle appearing in any given document is low. Depends on collection.
- Hashing: hash k-shingles instead of using them directly in algorithms that follow
- Using words, effective for similarity (more meaning) but sparser, set of possible shingles is huge

Min-Hash

Clever idea: let's summarize item(sets) (reduce data size!) but make it easy to find similar item(sets).

Min-Hash

Characteristic Matrix

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

Min-Hash

- Permute the rows of the characteristic matrix
- Min-Hash value of set: first non-zero row in corresponding column

Min-Hash

Permuted characteristic Matrix

Element	S_1	S_2	S_3	S_4
b	0	0	1	0
e	0	0	1	0
a	1	0	0	1
d	1	0	1	1
c	0	1	0	1

Min-Hash

Permuted characteristic Matrix

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b	0	0	1	0
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$$\mathrm{min\_hash}(S_1) = a, \mathrm{min\_hash}(S_2) = c$$

Min-Hash

Property: $Pr(\text{min\_hash}(S_i) = \text{min\_hash}(S_j)) = JS(S_i, S_j)$

Pf: on the board

Min-Hash Signatures

- Choose n permutations of rows, and set $h_i(S_j)$ as the Min-Hash given by permutation i of set j
- Represent set j by the *signature* vector of Min-hashes $[h_1(S_j), \ldots, h_n(S_j)]$
- Collect signature vectors into a *signature matrix*

Min-Hash Signatures in Practice

Instead of row permutations, use hash functions h_i over row indices

Let $SIG(i, c)$ be the ith hash of cth element

Initialize: set $SIG(i, c) = \infty$ for all i and c
Row r:

- Compute $h_i(r)$ for all i
- For each column c:
  - If c has a 0 in row r, do nothing
  - If c has a 1 in row r, then for each $i = 1, \ldots, n$:
    set $SIG(i, c)$ to $\min(SIG(i, c), h_i(r))$

Exercise

Element	S_1	S_2	S_3	S_4
0	0	1	0	1
1	0	1	0	0
2	1	0	0	1
3	0	0	1	0
4	0	0	1	1
5	1	0	0	0

$h_1(x) = 2x + 1 \mod 6$ $h_2(x) = 3x + 2 \mod 6$
$h_3(x) = 5x + 2 \mod 6$

JS and Minhashing

Estimate $JS(S_i, S_j)$ as the proportion of rows (hashes) of the signature matrix that are equal for columns S_i and S_j.

Exercise

Prove that if the JS of two sets is 0, then Min-Hash always gives the right answer.

Locality-Sensitive Hashing

Minhash gives a compressed representation of item(sets) that retains similarity

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But to find all pairs of similar item(sets) can still take a lot of time

LSH gives us a way of only comparing likely similar pairs.
Conversely, ignore pairs that are unlikely similar

LSH for Minhash

- Divide signature matrix into b bands, each with r rows
- For each column (itemset) and band, hash it's r entries according to some hash function
- Use same hash function in each of the bands, but use different hash arrays

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Itemsets with similar signatures will hash to the same bucket with some likelihood (candidate similar pair)

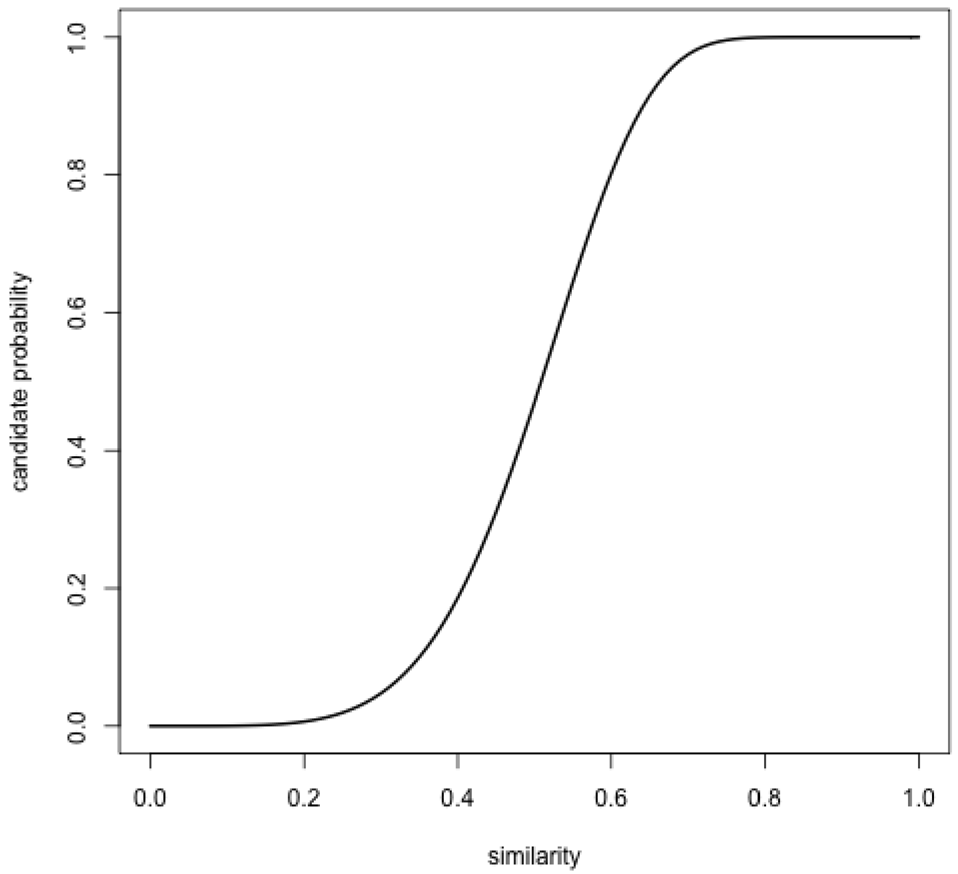
Itemsets without matching signatures will not

Analysis of LSH

Let $JS(S,T) = s$

- Probability signatures agree in all rows of one band: s^r
- Probability do not agree in at least one row of a band: $1 - s^r$
- Probability that signatures do not agree in all rows of any of the bands: $(1 - s^r)^b$
- Probability that signatures agree in all the rows of at least one band (hash to the same bucket at least once): $1 - (1 - s^r)^b$.

Analysis of LSH



Final algorithm for similar document search

Part I: Shingles

- Pick a value of k, construct k-shingles for each document (optionally hashing k-shingles)
- Sort documents by document-shingle pairs by shingle

Final algorithm for similar document search

Part II: Minhash

- Pick a length n for minhash signatures
- Compute minhash signatures for all documents

Final algorithm for similar document search

Part III: LSH

- Choose threshold t for how similar documents have to be to consider as a similar pair
- Choose number of bands b and number of rows r such that $br = n$ and threshold t is approximately $(1/b)^{(1/r)}$
- Construct candidate pairs using LSH

Final algorithm for similar document search

Part IV: Confirm similar pairs

- For each candidate pair, confirm that their signatures match in at least t fraction of rows
- Optionally, verify similarity in shingled documents

Frequent Itemsets

Find items that occur frequently together in sets

Examples:

- items frequently bought together in the same transaction
- words that appear frequently together in the same document

Market-Basket Model

Items: objects we are modeling

Baskets: sets of items (transactions)

Frequent itemsets: items that co-occur frequently in baskets

Frequent Itemsets

$\mathrm{support}(I)$ the number of baskets in which itemset I appears

Frequent itemsets: Itemsets I with $\mathrm{support}(I) \geq s$ (for some threshold s)

Example

(1) {Cat, and, dog, bites}

(2) {Yahoo, news, claims, a, cat, mated, with, a, dog, and, produced, viable, offspring}

(3) {Cat, killer, likely, is, a, big, dog}

(4) {Professional, free, advice, on, dog, training, puppy, training}

(5) {Cat, and, kitten, training, and, behavior}

(6) {Dog, &, Cat, provides, dog, training, in, Eugene, Oregon}

(7) {Shop, for, your, show, dog, grooming, and, pet, supplies}

For $s = 3$ what are frequent itemsets of size 1 or 2

Exercise

Suppose there are 100 items, numbered 1 to 100, and also 100 baskets, numbered 1 to 100.

Item i is in basket B iff i divides B with no remainder

- Item 1 is in all baskets, item 2 in the even-numbered baskets
- Basket 24 contains items {1,2,3,4,6,8,12,24}

a) If support threshold is 5, which items are frequent? b) Which pairs are frequent?

Association Rules

Rules of the form `I -> j`: if itemset I is in basket, then item j is likely in basket as well

rule confidence: $\text{confidence}(I \rightarrow j) = \frac{\text{support}(I \bigcup \{j\})}{\text{support}(I)}$

rule interest: $\text{confidence}(I \rightarrow j) - |\{B \in \text{Baskets s.t. } j \in B\}|$

Association Rules

Note: once we have frequent itemsets, we can get association rules with high confidence easily

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Suppose we find **all** frequent itemsets over some support threshold s

Let itemset J with n items be one of those itemsets ($\mathrm{support}(J) \geq s$), then

1. there are only n candidate Association Rules $(J - \{j\}) \rightarrow j$
2. $\mathrm{support}(J) \geq s$ implies $\mathrm{support}(J - \{j\}) \geq s$ so we must have already calculated the latter
3. We can quickly compute the *confidence* and *interest* of each rule

Itemset monotonicity

If I is a frequent itemset, then every subset of I is a frequent itemset

Why?

The A-priori algorithm

Suppose we are given a set of baskets over n items

First pass

Count the number of occurrences of each item (array of n values)

After first pass

Identify frequent singletons (above support threshold)

The A-priori algorithm

Second pass

Count the number of occurrences of pairs of frequent items

- For each basket:
  - Check which of its items are frequent (first pass)
  - For each pair of items increase occurrence count

After the second pass

Identify frequent pairs (above support threshold)

The A-priori algorithm

Third pass

Count the number of occurrences of frequent pairs + a frequent item

- For each basket:
  - Check which item pairs and singletons that are frequent (first and second pass)
  - For each combination of pair and singleton, increase occurrence count

After the third pass

Identify frequent triples (above support threshold)

The A-priori algorithm

And so on until no more frequent sets are identified

Notes:

- The data structure to store pair counts will be important consideration
- The algorithm has a construct-filter structure: at each pass, *construct* the set of candidate itemsets, *filter* to those that are frequent

Exercise

Apply A-priori algorithm to previous exercise

Handling large datasets

For large datasets storing occurrences of candidate frequent pairs is problematic

PCY algorithm: hash item pairs and keep count in hash bucket

Define candidate frequent pairs as

- i and j are frequent items
- $\{i, j\}$ hashes to a frequent bucket (with count > threshold)

Handling large datasets

Identify frequent buckets with a bitmap (little memory)

Only count (and verify) candidate pairs as defined above (expected to be much fewer)

Exercise

Consider baskets over items $1, \ldots, 6$

$$\{1,2,3\}\ \{2,3,4\}\ \{3,4,5\}\ \{4,5,6\}$$
$$\{1,3,5\}\ \{2,4,6\}\ \{1,3,4\}\ \{2,4,5\}$$
$$\{3,5,6\}\ \{1,2,4\}\ \{2,3,5\}\ \{3,4,6\}$$

- Compute support for each item and each pair of items
- Using hash function $i \times j \mod 11$ (hash table with 11 buckets), which pairs hash to the same buckets?

Exercise

- Which buckets are frequent?
- Which pairs are counted in the second pass of PCY algorithm?

Summary

Itemset analysis: applications to collaborative filtering, recommendation engines

Finding Similar Itemsets

- Jaccard similarity: measure of set similarity based on common items
- Minhashing with LSH: effective way of finding similar itemsets with efficient data structures for large datasets

Summary

Finding Frequent Itemsets

- Market-basket data: model of item transactions
- Frequent Itemsets: Sets of items appearing frequently in "baskets"
- Association Rules: $I \to j$
- Pair-counting Bottleneck: frequent itemset mining memory space taken mostly in keeping counts of pairs of frequent items
- Monotonicity of frequent itemsets
- A-priori Algorithm
- Hashing for large datasets