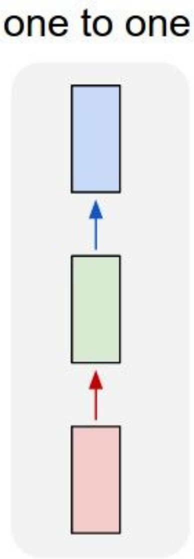
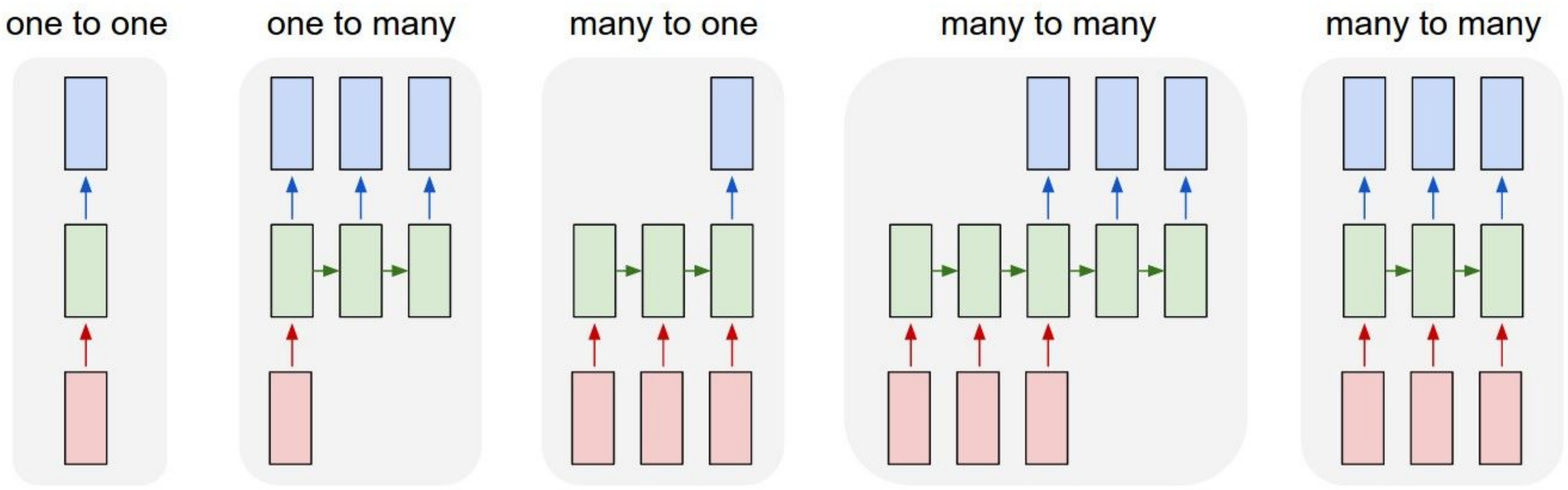
“Vanilla” Neural Network



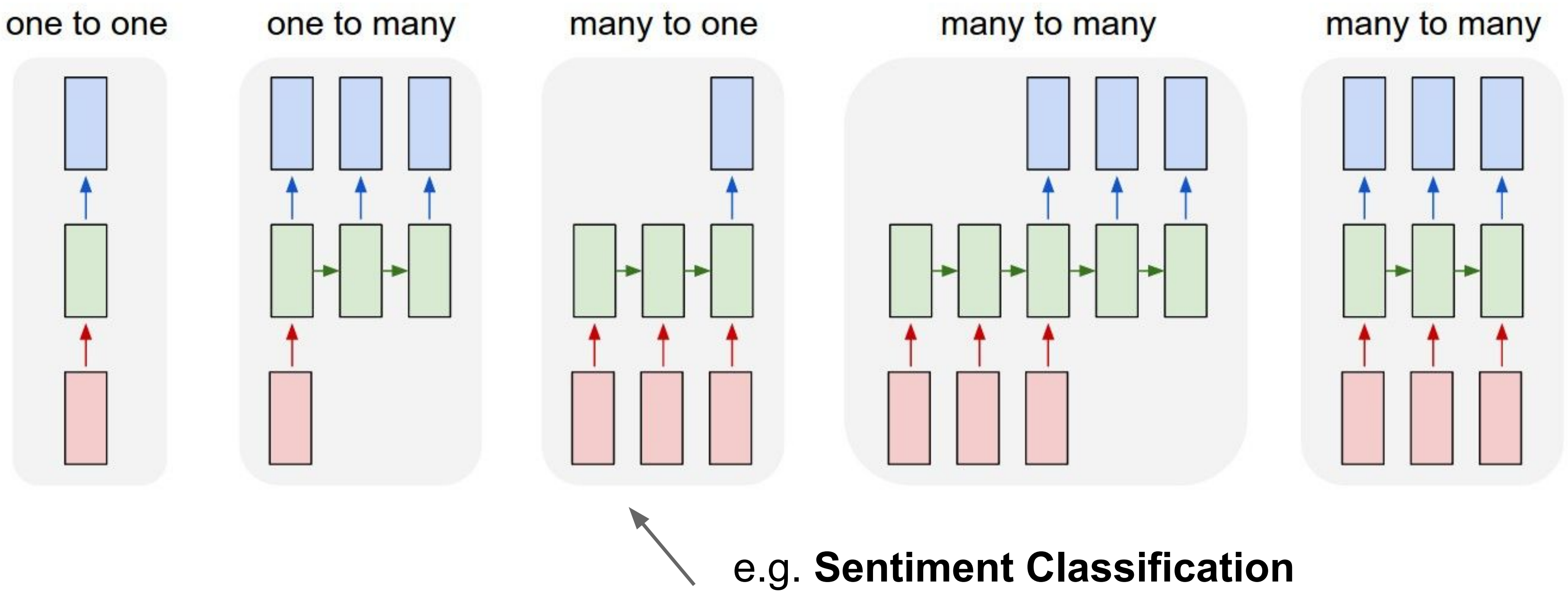
Vanilla Neural Networks

Recurrent Neural Networks: Process Sequences



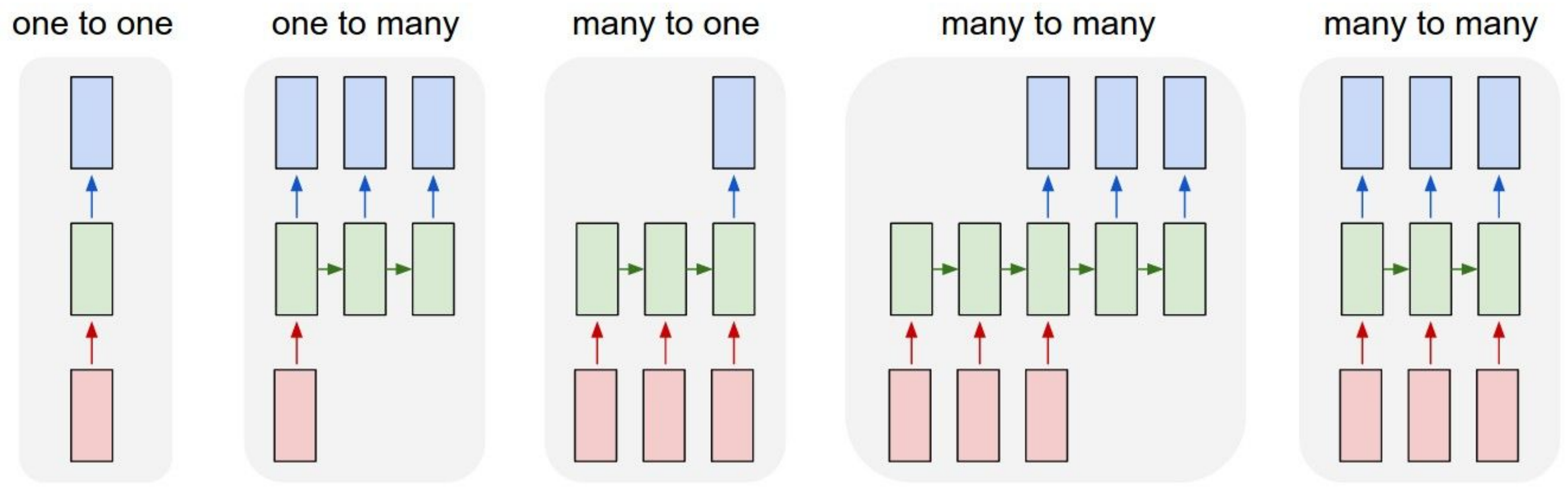
e.g. **Image Captioning**
image -> sequence of words

Recurrent Neural Networks: Process Sequences



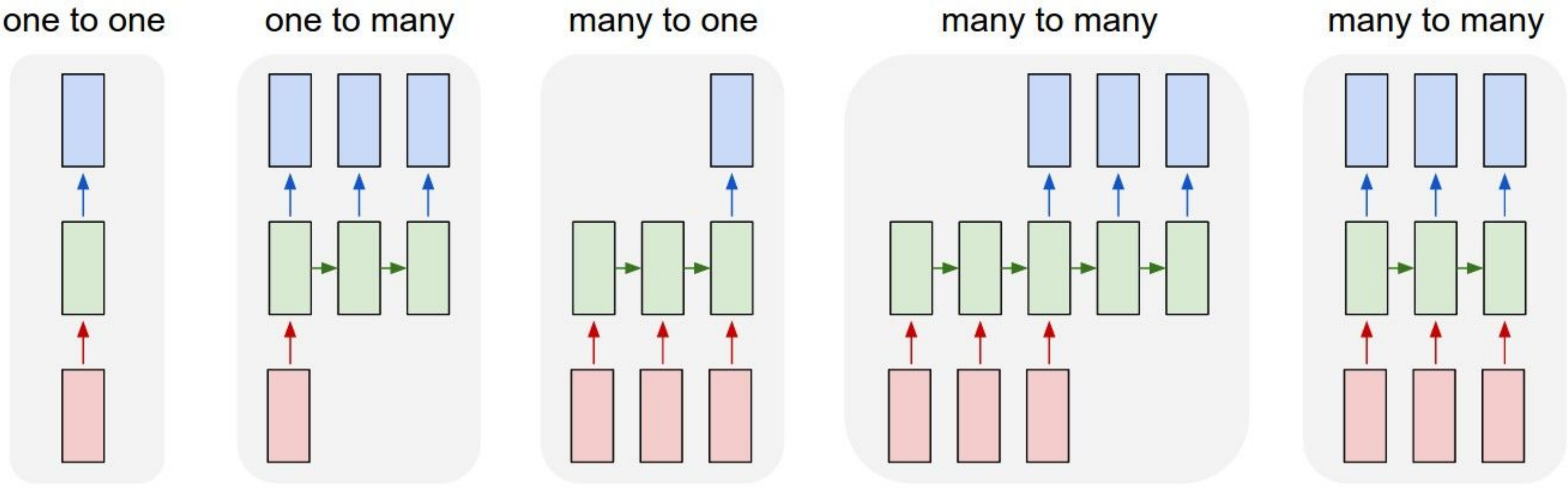
e.g. **Sentiment Classification**
sequence of words -> sentiment

Recurrent Neural Networks: Process Sequences



e.g. **Machine Translation**
seq of words -> seq of words

Recurrent Neural Networks: Process Sequences



e.g. **Video classification on frame level**

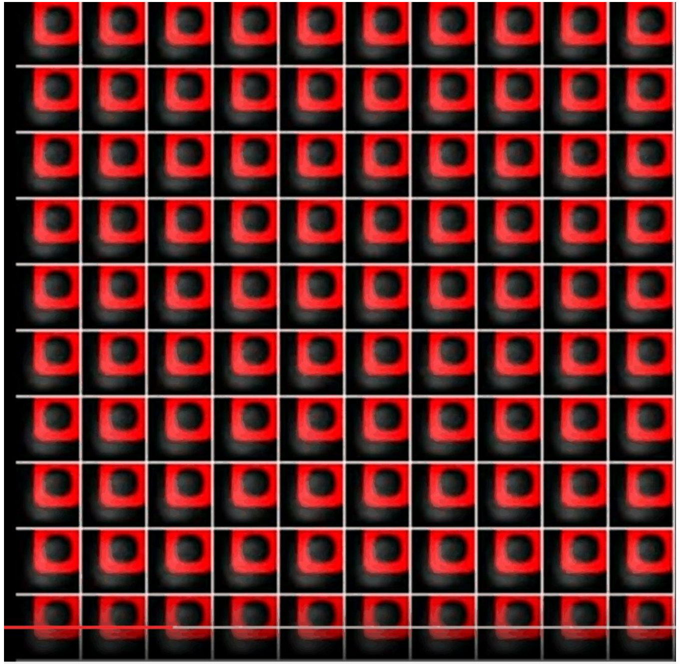
Sequential Processing of Non-Sequence Data

Classify images by taking a series of "glimpses"

Ba, Mnih, and Kavukcuoglu, "Multiple Object Recognition with Visual Attention", ICLR 2015.
Gregor et al, "DRAW: A Recurrent Neural Network For Image Generation", ICML 2015

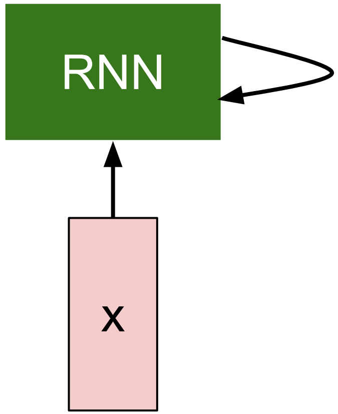
Sequential Processing of Non-Sequence Data

Generate images one piece at a time!

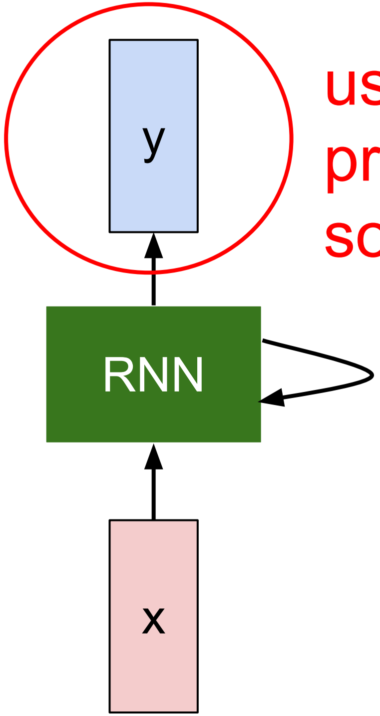


Gregor et al, "DRAW: A Recurrent Neural Network For Image Generation", ICML 2015

Recurrent Neural Network



Recurrent Neural Network



usually want to predict a vector at some time steps

Recurrent Neural Network

We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

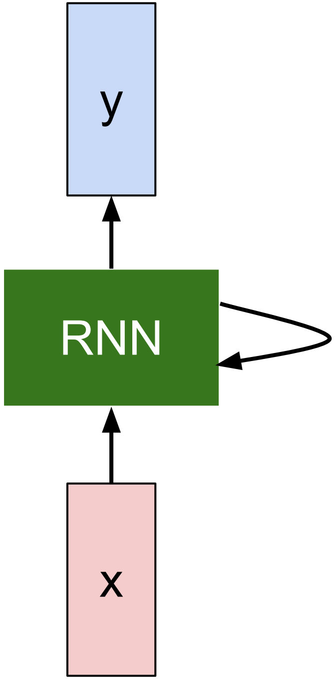
$$\underbrace{\boxed{h_t}}_{\text{new state}} = \boxed{f_W}(\underbrace{\boxed{h_{t-1}}}_{\text{old state}}, \underbrace{\boxed{x_t}}_{\text{input vector at some time step}})$$

new state

old state

input vector at some time step

some function with parameters W

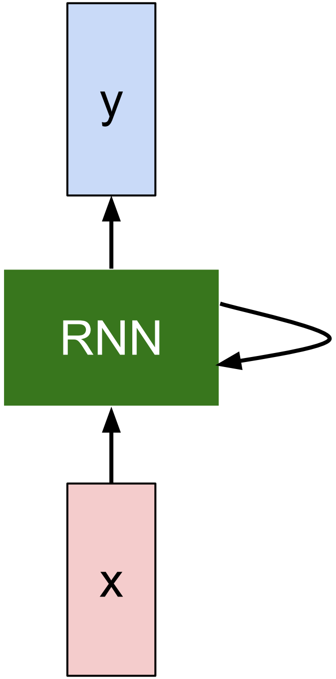


Recurrent Neural Network

We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

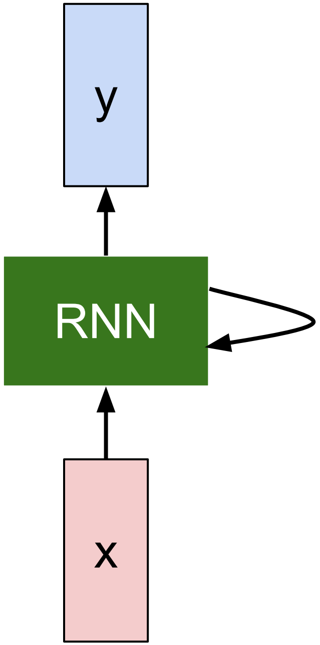
$$h_t = f_W(h_{t-1}, x_t)$$

Notice: the same function and the same set of parameters are used at every time step.



(Vanilla) Recurrent Neural Network

The state consists of a single *"hidden"* vector **h**:

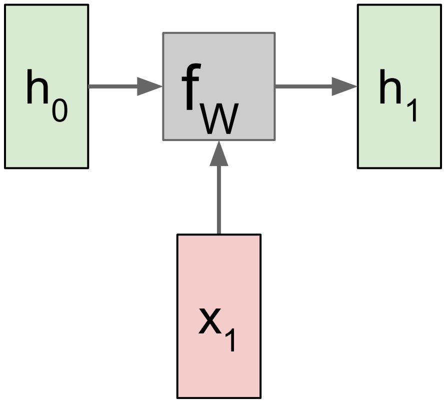


$$h_t = f_W(h_{t-1}, x_t)$$

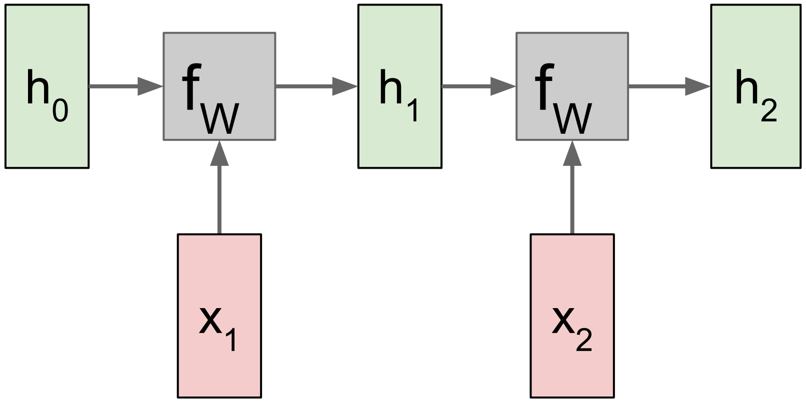
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

$$y_t = W_{hy}h_t$$

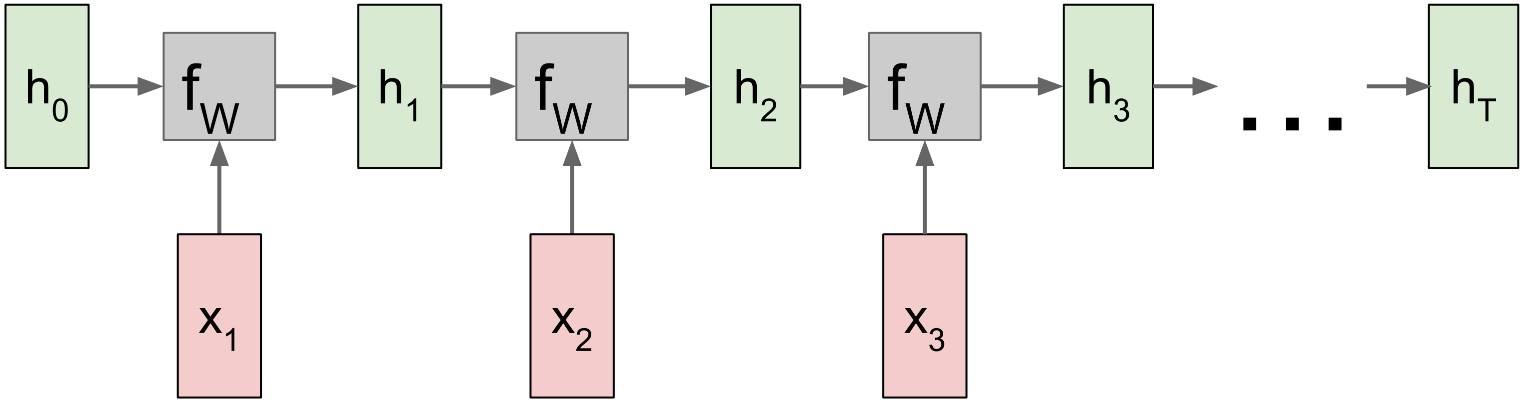
RNN: Computational Graph



RNN: Computational Graph

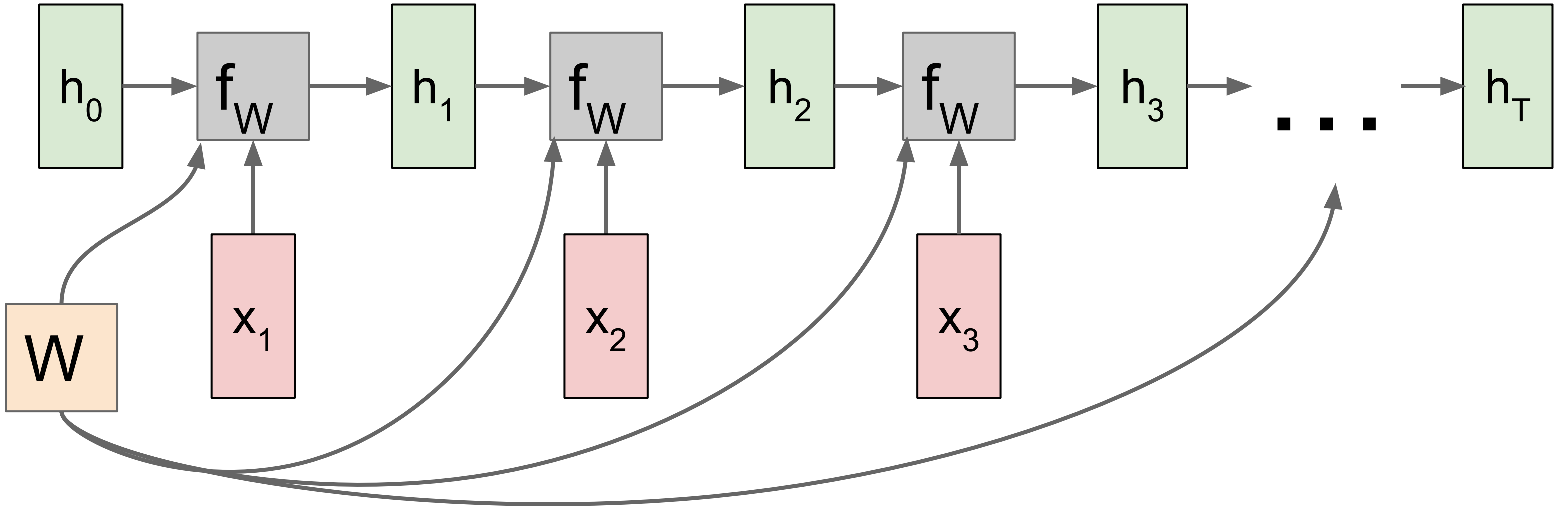


RNN: Computational Graph

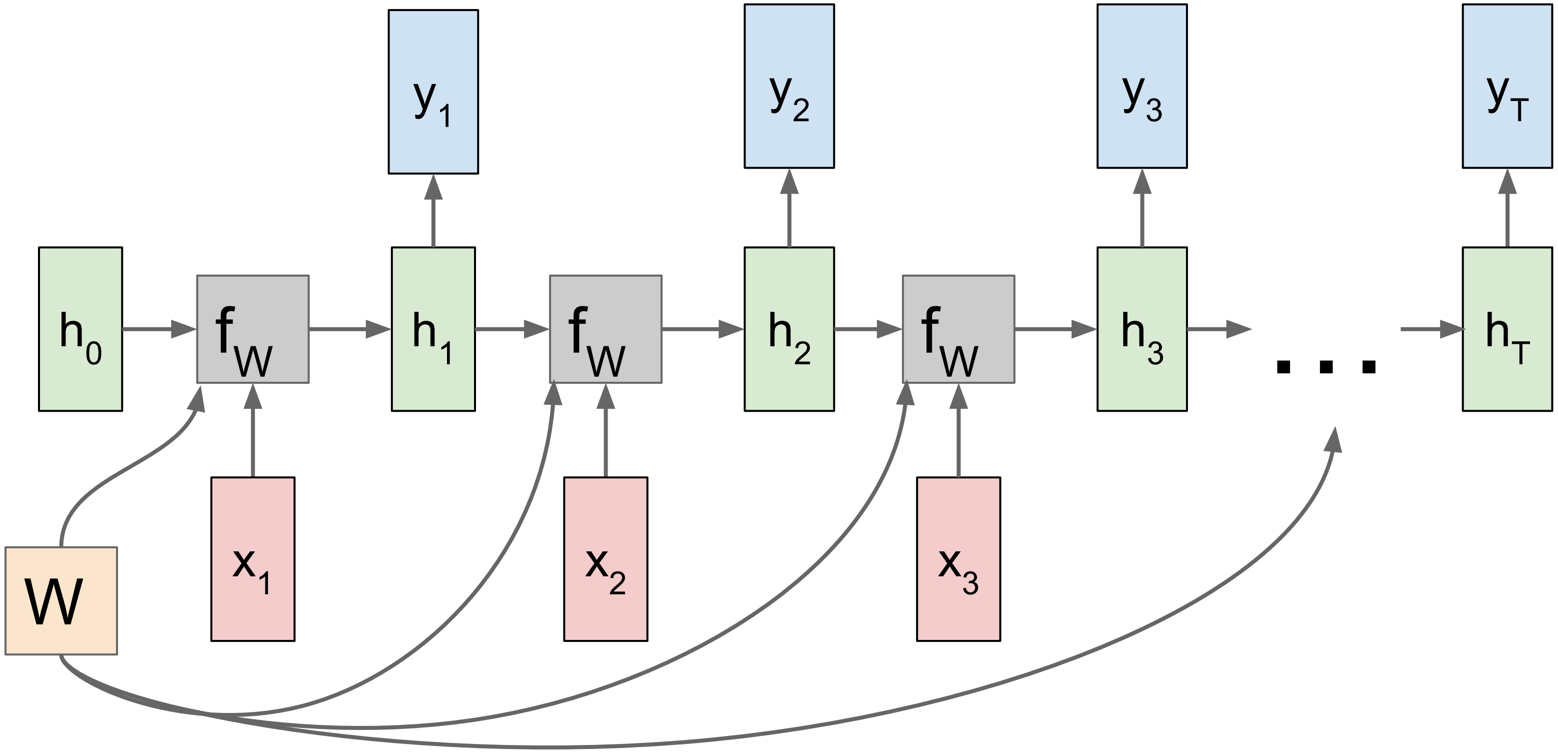


RNN: Computational Graph

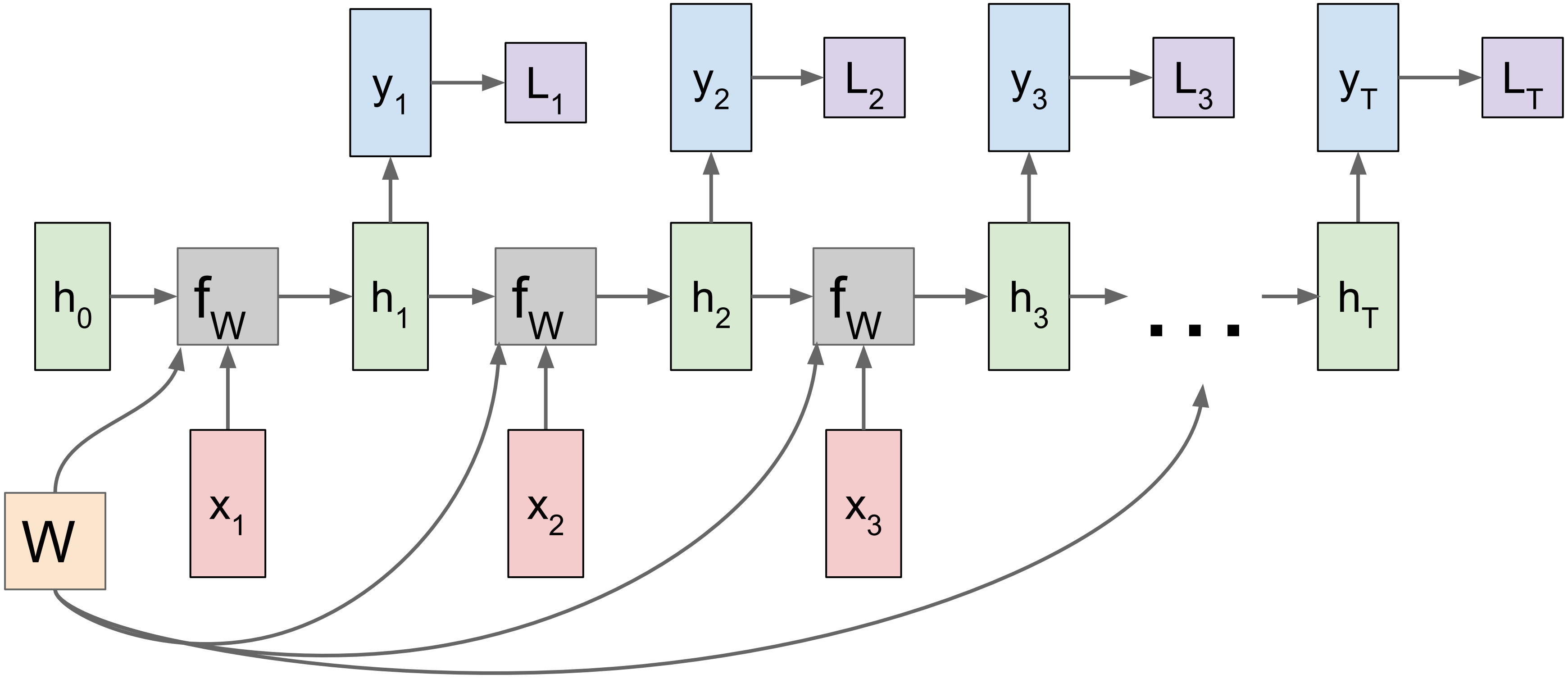
Re-use the same weight matrix at every time-step



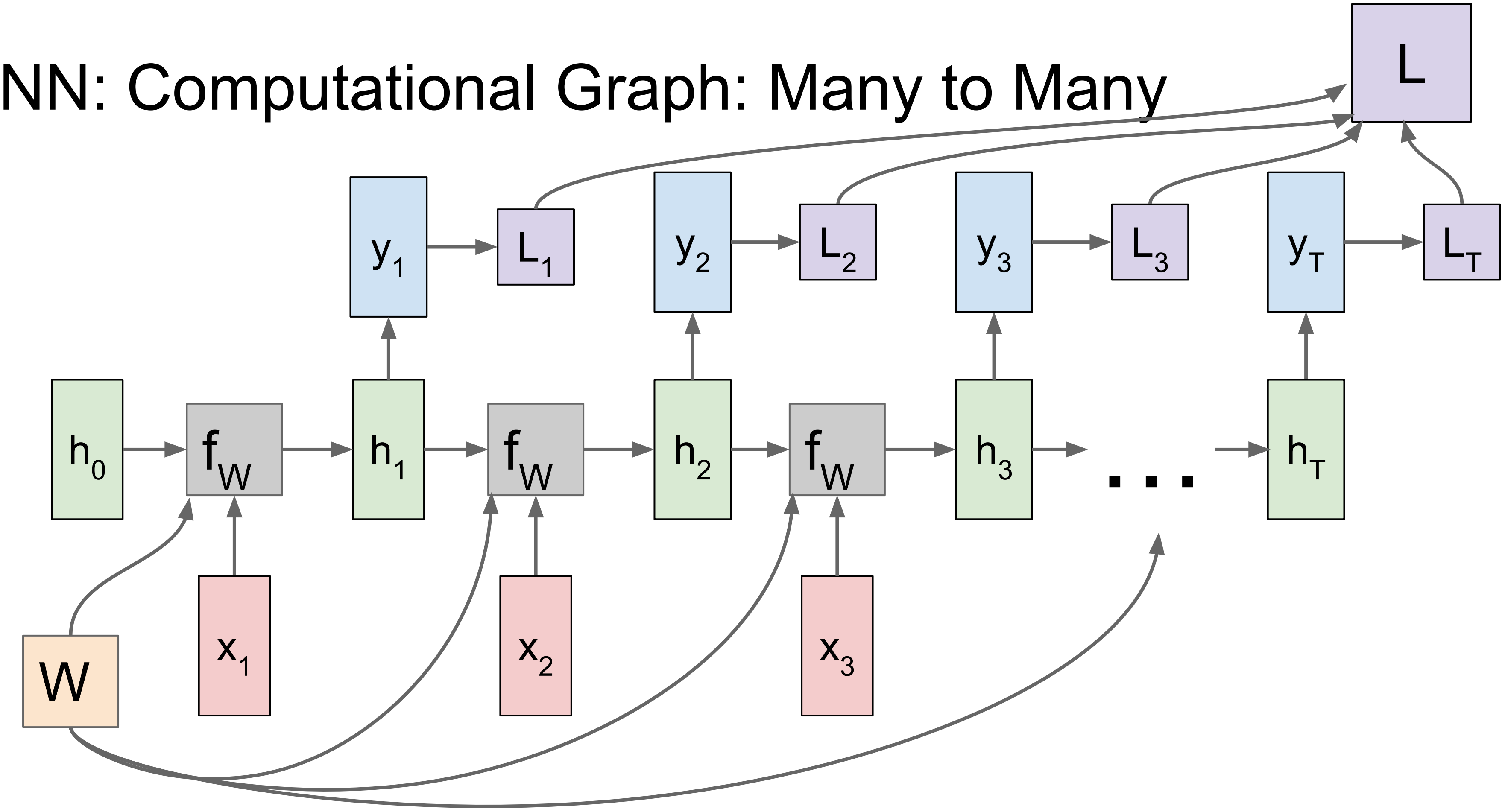
RNN: Computational Graph: Many to Many



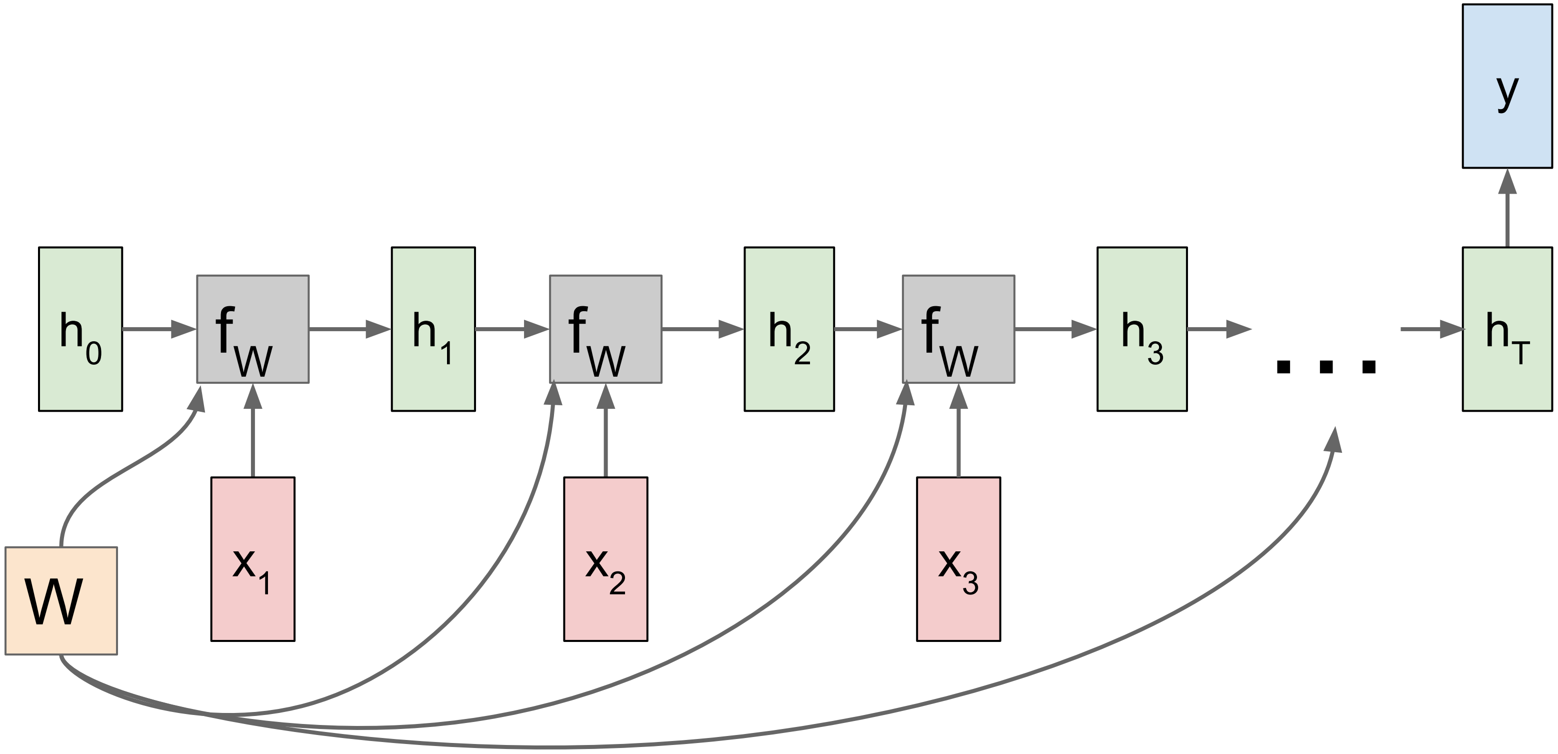
RNN: Computational Graph: Many to Many



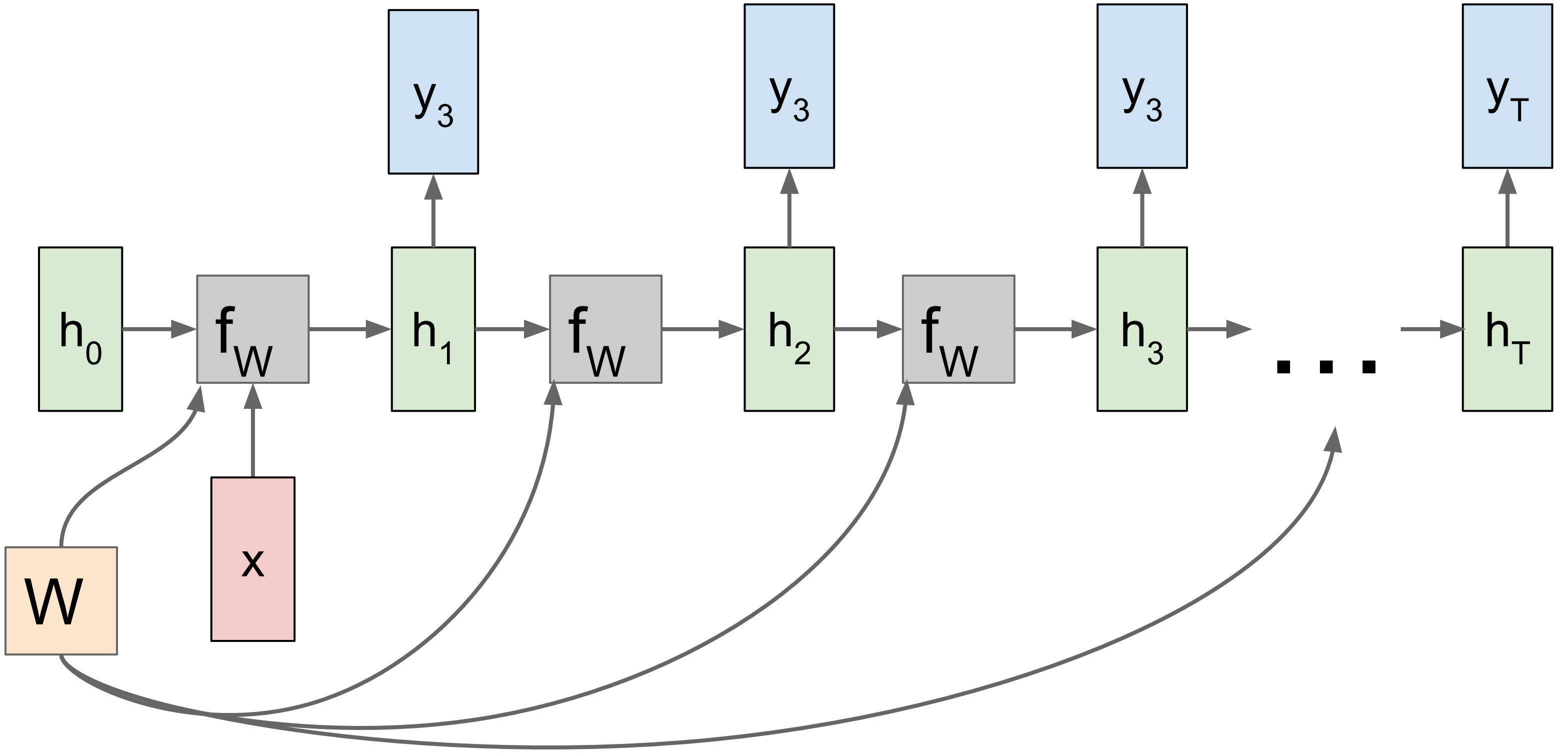
RNN: Computational Graph: Many to Many



RNN: Computational Graph: Many to One

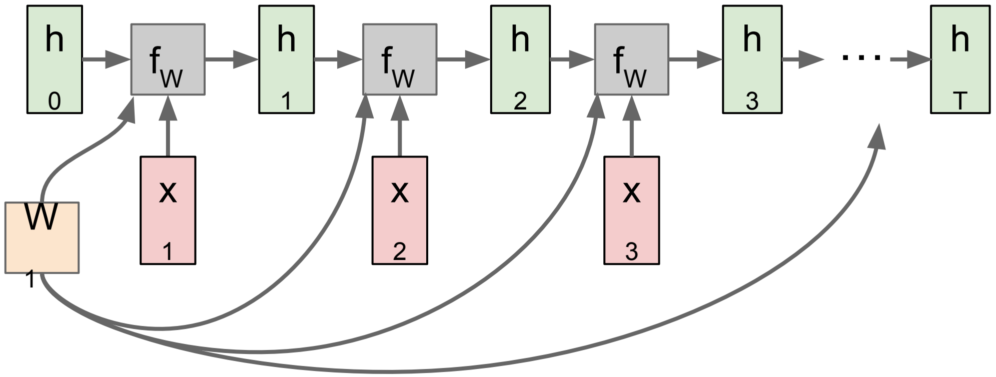


RNN: Computational Graph: One to Many



Sequence to Sequence: Many-to-one + one-to-many

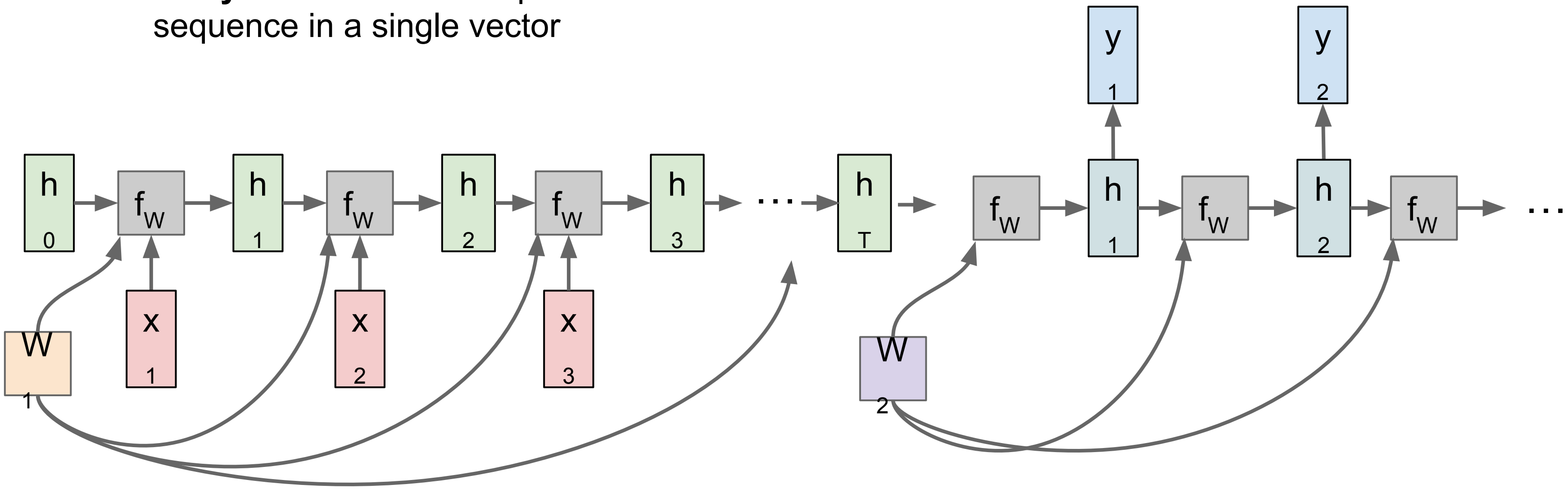
Many to one: Encode input sequence in a single vector



Sequence to Sequence: Many-to-one + one-to-many

One to many: Produce output sequence from single input vector

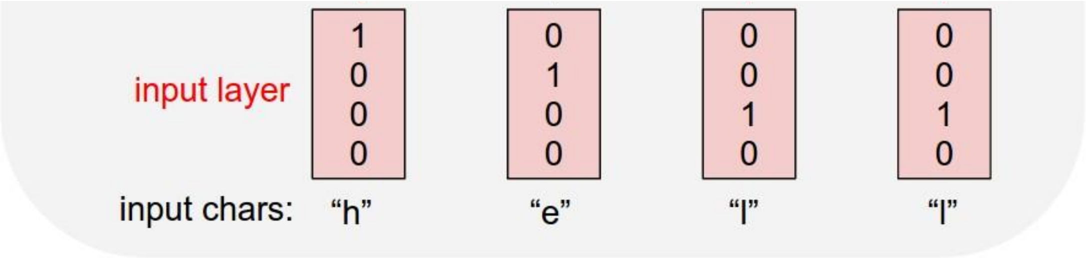
Many to one: Encode input sequence in a single vector



Example:
Character-level
Language Model

Vocabulary:
[h,e,l,o]

Example training sequence:
“hello”

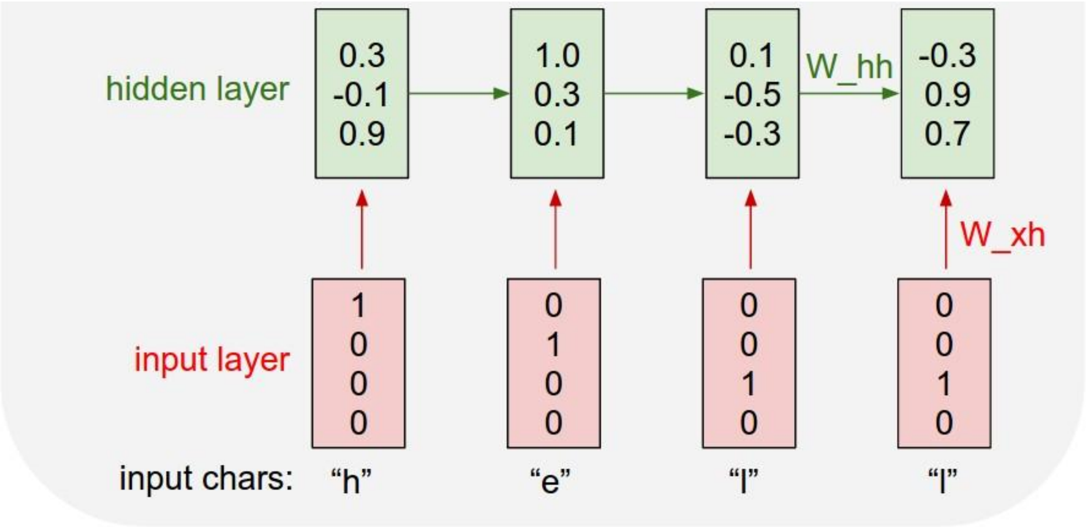


Example: Character-level Language Model

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Vocabulary: [h,e,l,o]

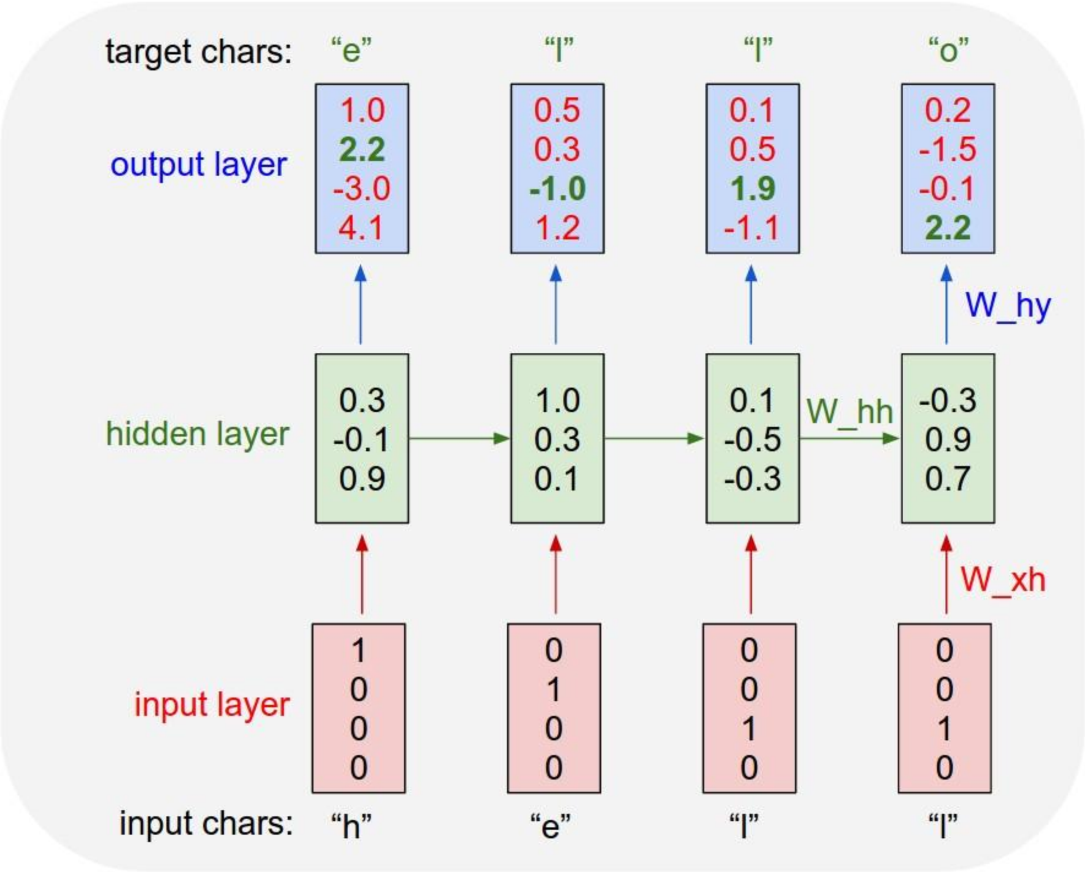
Example training sequence: **“hello”**



Example: Character-level Language Model

Vocabulary:
[h,e,l,o]

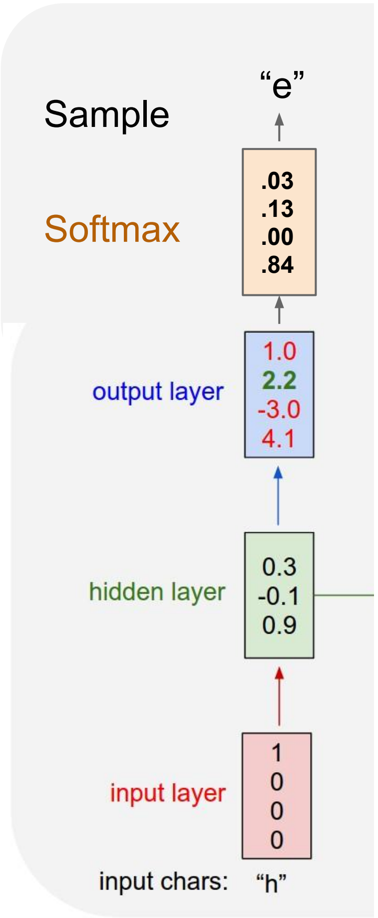
Example training sequence:
“hello”



Example: Character-level Language Model Sampling

Vocabulary: [h,e,l,o]

At test-time sample characters one at a time, feed back to model



Example: Character-level Language Model Sampling

Vocabulary:
[h,e,l,o]

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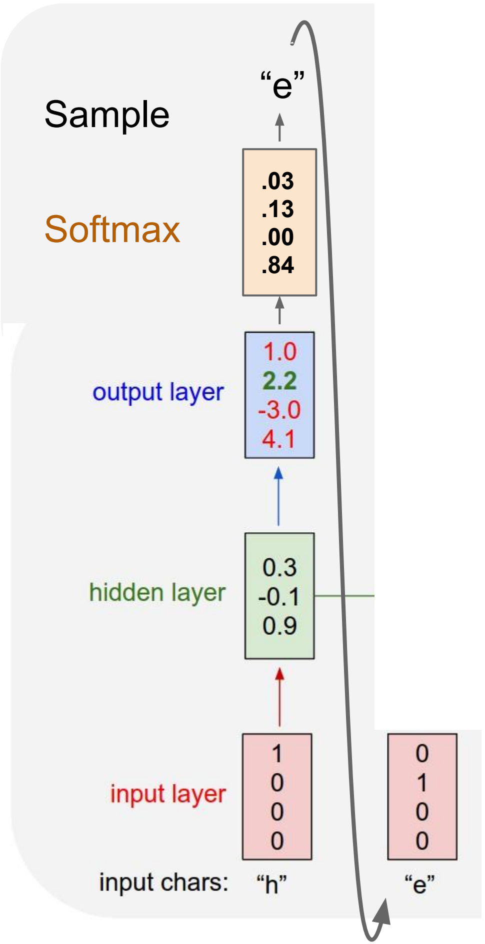

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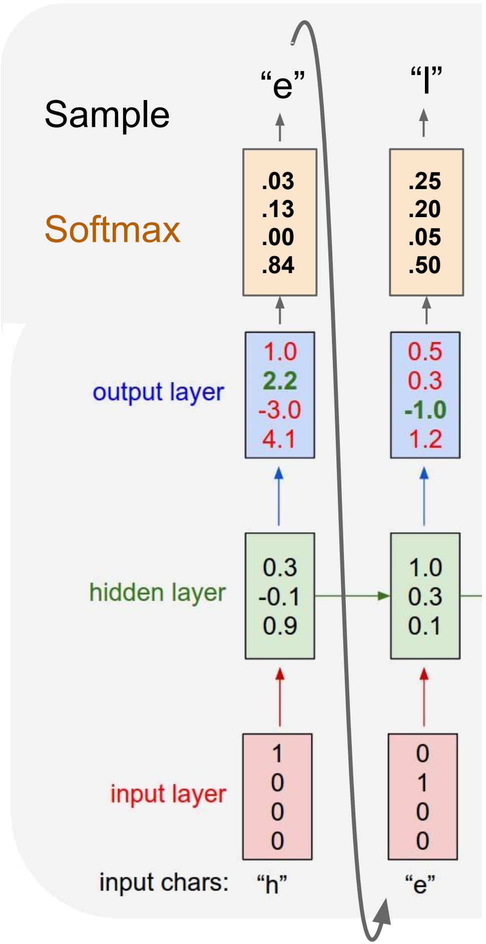

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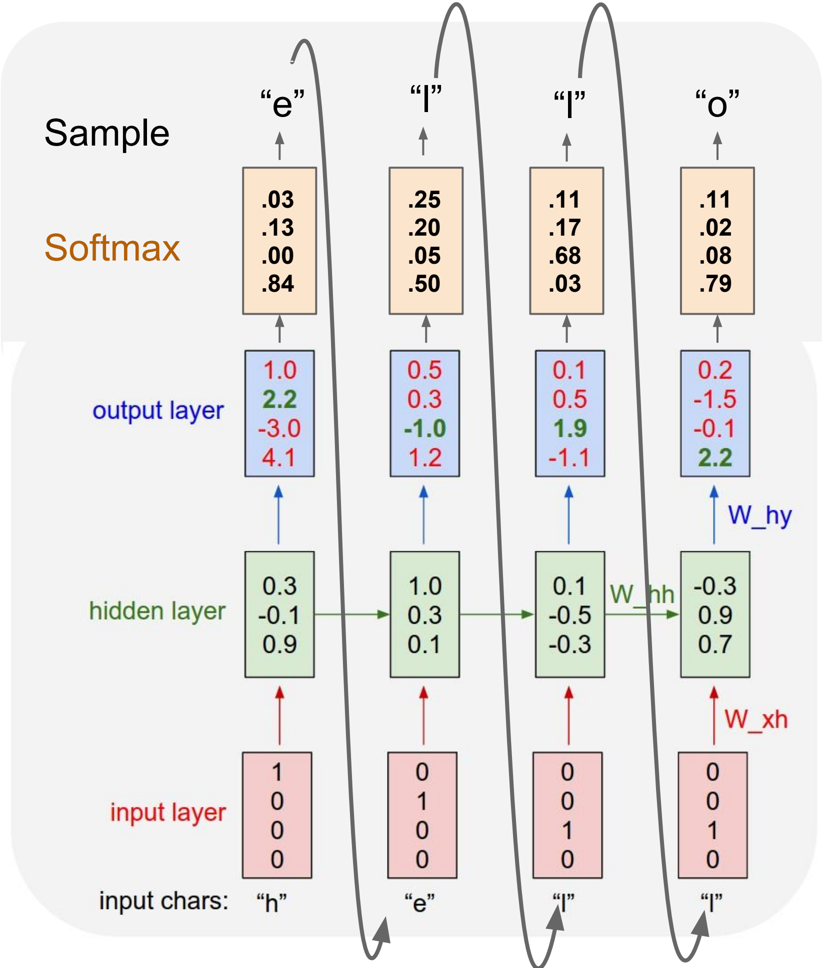
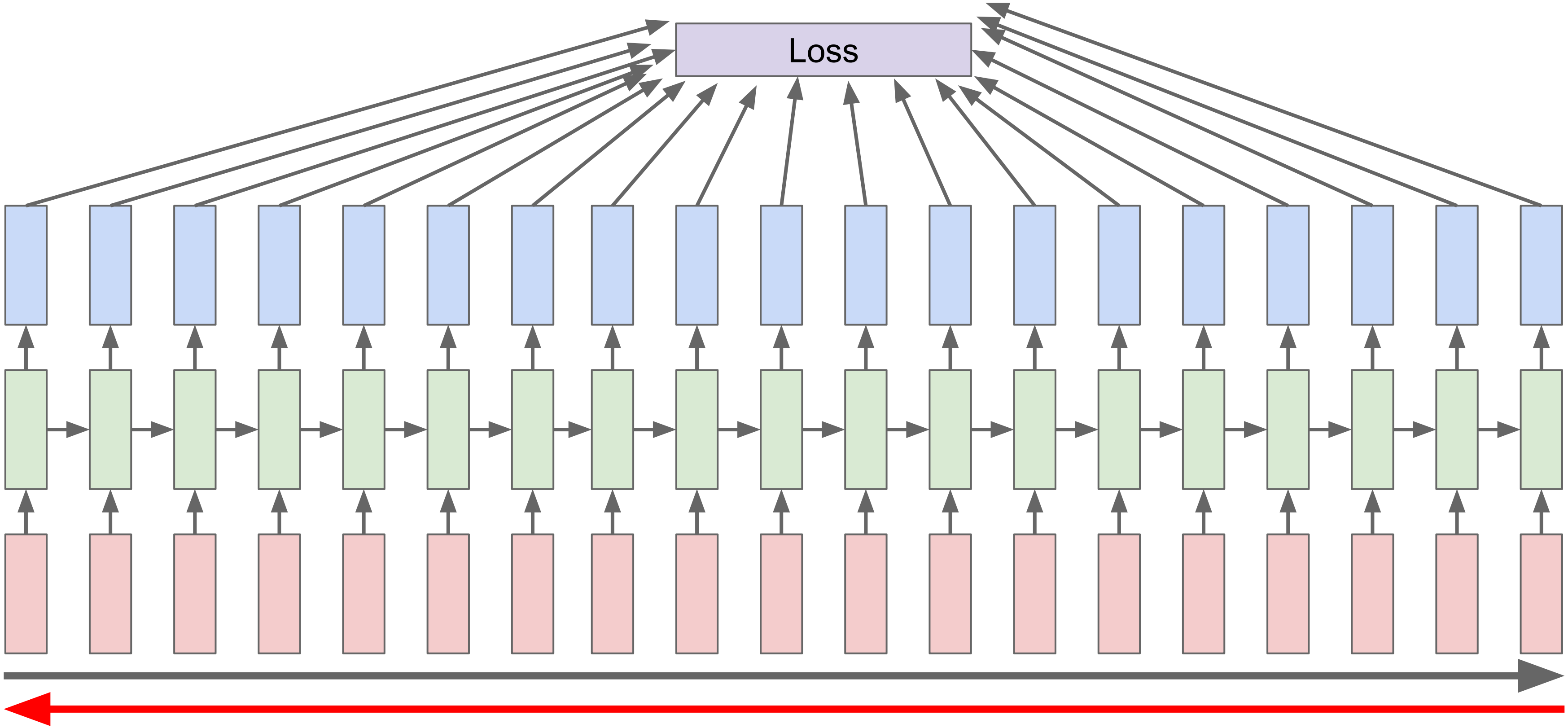
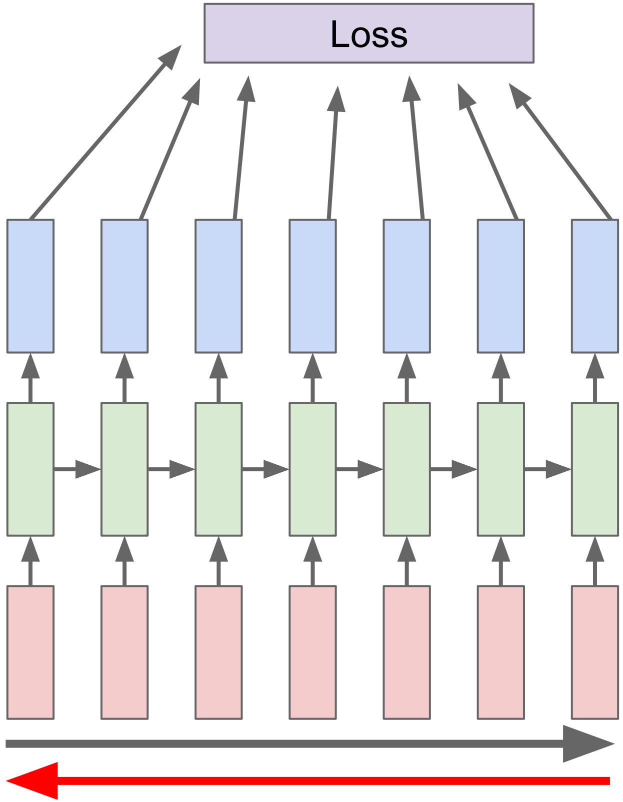


Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

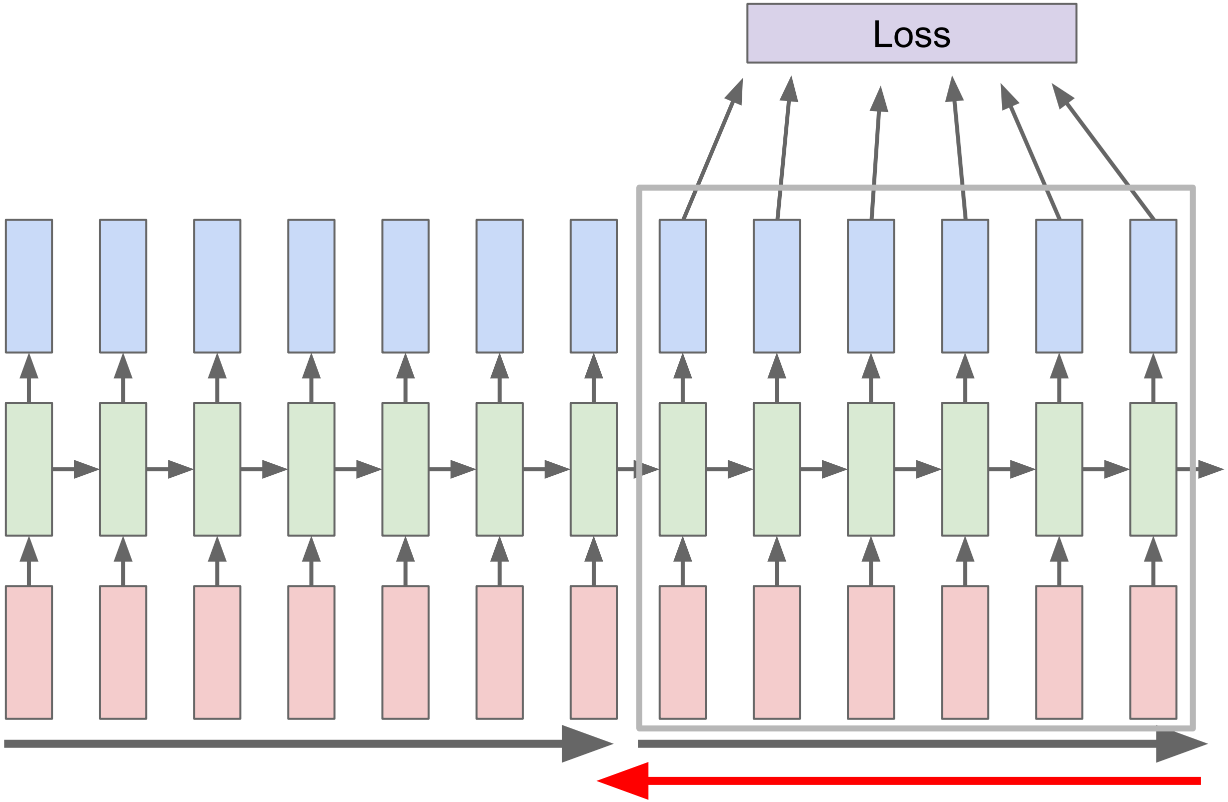


Truncated Backpropagation through time



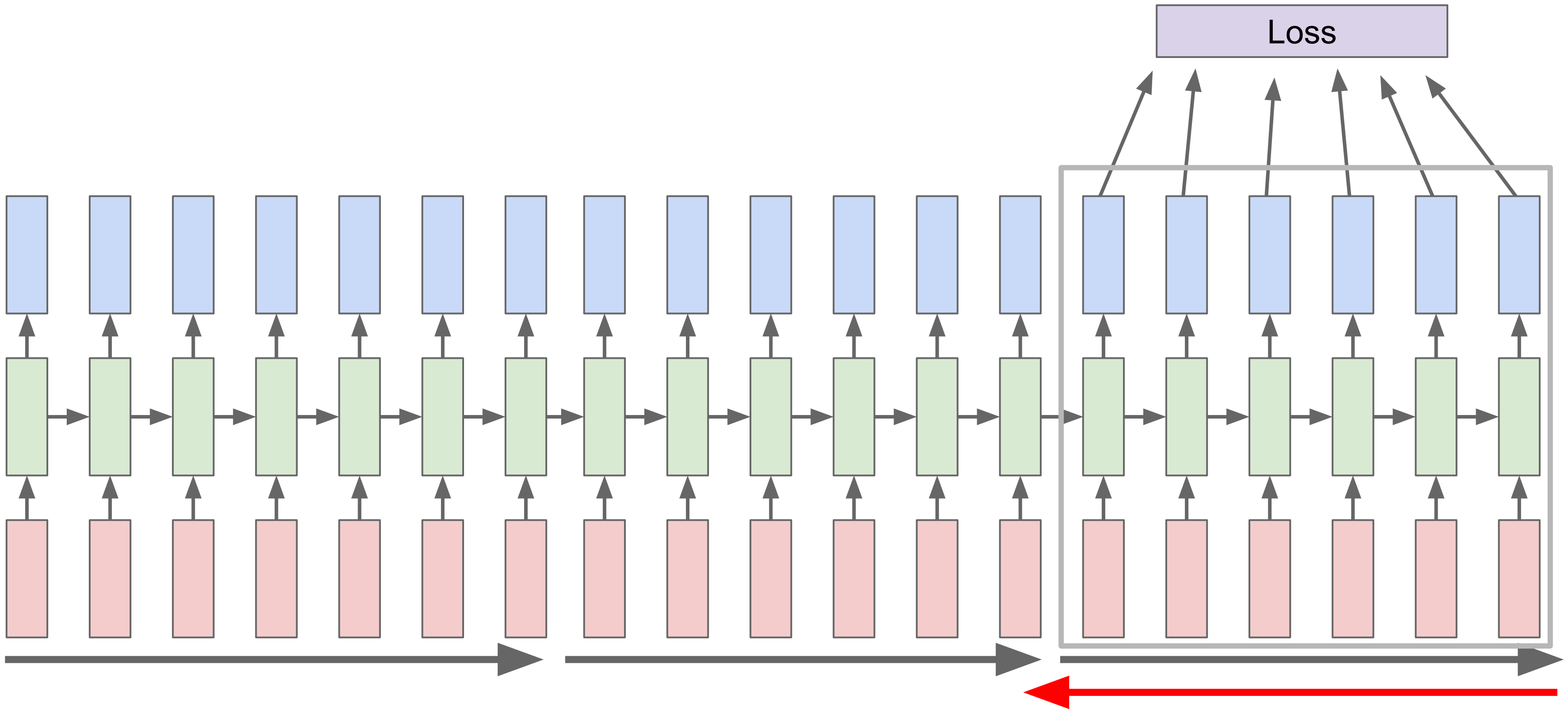
Run forward and backward through chunks of the sequence instead of whole sequence

Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Truncated Backpropagation through time



min-char-rnn.py gist: 112 lines of Python

```python
"""
Minimal character-level Vanilla RNN model. Written by Andrej Karpathy (@karpathy)
BSD License
"""
import numpy as np

# data I/O
data = open('input.txt', 'r').read() # should be simple plain text file
chars = list(set(data))
data_size, vocab_size = len(data), len(chars)
print 'data has %d characters, %d unique.' % (data_size, vocab_size)
char_to_ix = { ch:i for i,ch in enumerate(chars) }
ix_to_char = { i:ch for i,ch in enumerate(chars) }

# hyperparameters
hidden_size = 100 # size of hidden layer of neurons
seq_length = 25 # number of steps to unroll the RNN for
learning_rate = 1e-1

# model parameters
Wxh = np.random.randn(hidden_size, vocab_size)*0.01 # input to hidden
Whh = np.random.randn(hidden_size, hidden_size)*0.01 # hidden to hidden
Why = np.random.randn(vocab_size, hidden_size)*0.01 # hidden to output
bh = np.zeros((hidden_size, 1)) # hidden bias
by = np.zeros((vocab_size, 1)) # output bias

def lossFun(inputs, targets, hprev):
  """
  inputs,targets are both list of integers.
  hprev is Hx1 array of initial hidden state
  returns the loss, gradients on model parameters, and last hidden state
  """
  xs, hs, ys, ps = {}, {}, {}, {}
  hs[-1] = np.copy(hprev)
  loss = 0
  # forward pass
  for t in xrange(len(inputs)):
    xs[t] = np.zeros((vocab_size,1)) # encode in 1-of-k representation
    xs[t][inputs[t]] = 1
    hs[t] = np.tanh(np.dot(Wxh, xs[t]) + np.dot(Whh, hs[t-1]) + bh) # hidden state
    ys[t] = np.dot(Why, hs[t]) + by # unnormalized log probabilities for next chars
    ps[t] = np.exp(ys[t]) / np.sum(np.exp(ys[t])) # probabilities for next chars
    loss += -np.log(ps[t][targets[t],0]) # softmax (cross-entropy loss)
  # backward pass: compute gradients going backwards
  dWxh, dWhh, dWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
  dbh, dby = np.zeros_like(bh), np.zeros_like(by)
  dhnext = np.zeros_like(hs[0])
  for t in reversed(xrange(len(inputs))):
    dy = np.copy(ps[t])
    dy[targets[t]] -= 1 # backprop into y
    dWhy += np.dot(dy, hs[t].T)
    dby += dy
    dh = np.dot(Why.T, dy) + dhnext # backprop into h
    dhraw = (1 - hs[t] * hs[t]) * dh # backprop through tanh nonlinearity
    dbh += dhraw
    dWxh += np.dot(dhraw, xs[t].T)
    dWhh += np.dot(dhraw, hs[t-1].T)
    dhnext = np.dot(Whh.T, dhraw)
  for dparam in [dWxh, dWhh, dWhy, dbh, dby]:
    np.clip(dparam, -5, 5, out=dparam) # clip to mitigate exploding gradients
  return loss, dWxh, dWhh, dWhy, dbh, dby, hs[len(inputs)-1]
```

```python
def sample(h, seed_ix, n):
  """
  sample a sequence of integers from the model
  h is memory state, seed_ix is seed letter for first time step
  """
  x = np.zeros((vocab_size, 1))
  x[seed_ix] = 1
  ixes = []
  for t in xrange(n):
    h = np.tanh(np.dot(Wxh, x) + np.dot(Whh, h) + bh)
    y = np.dot(Why, h) + by
    p = np.exp(y) / np.sum(np.exp(y))
    ix = np.random.choice(range(vocab_size), p=p.ravel())
    x = np.zeros((vocab_size, 1))
    x[ix] = 1
    ixes.append(ix)
  return ixes

n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
while True:
  # prepare inputs (we're sweeping from left to right in steps seq_length long)
  if p+seq_length+1 >= len(data) or n == 0:
    hprev = np.zeros((hidden_size,1)) # reset RNN memory
    p = 0 # go from start of data
  inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
  targets = [char_to_ix[ch] for ch in data[p+1:p+seq_length+1]]

  # sample from the model now and then
  if n % 100 == 0:
    sample_ix = sample(hprev, inputs[0], 200)
    txt = ''.join(ix_to_char[ix] for ix in sample_ix)
    print '----\n %s \n----' % (txt, )

  # forward seq_length characters through the net and fetch gradient
  loss, dWxh, dWhh, dWhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
  smooth_loss = smooth_loss * 0.999 + loss * 0.001
  if n % 100 == 0: print 'iter %d, loss: %f' % (n, smooth_loss) # print progress

  # perform parameter update with Adagrad
  for param, dparam, mem in zip([Wxh, Whh, Why, bh, by],
                                [dWxh, dWhh, dWhy, dbh, dby],
                                [mWxh, mWhh, mWhy, mbh, mby]):
    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```

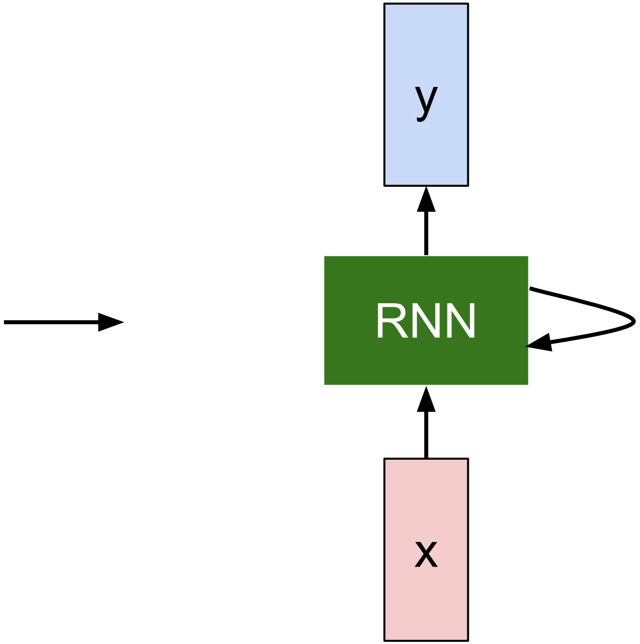
(https://gist.github.com/karpathy/d4dee566867f8291f086)

THE SONNETS

by William Shakespeare

From fairest creatures we desire increase,
That thereby beauty's rose might never die,
But as the riper should by time decease,
His tender heir might bear his memory:
But thou, contracted to thine own bright eyes,
Feed'st thy light's flame with self-substantial fuel,
Making a famine where abundance lies,
Thyself thy foe, to thy sweet self too cruel:
Thou that art now the world's fresh ornament,
And only herald to the gaudy spring,
Within thine own bud buriest thy content,
And tender churl mak'st waste in niggarding:
  Pity the world, or else this glutton be,
  To eat the world's due, by the grave and thee.

When forty winters shall besiege thy brow,
And dig deep trenches in thy beauty's field,
Thy youth's proud livery so gazed on now,
Will be a tatter'd weed of small worth held:
Then being asked, where all thy beauty lies,
Where all the treasure of thy lusty days;
To say, within thine own deep sunken eyes,
Were an all-eating shame, and thriftless praise.
How much more praise deserv'd thy beauty's use,
If thou couldst answer 'This fair child of mine
Shall sum my count, and make my old excuse,'
Proving his beauty by succession thine!
  This were to be new made when thou art old,
  And see thy blood warm when thou feel'st it cold.



at first:

```
tyntd-iafhatawiaoihrdemot  lytdws  e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e
plia tklrgd t o idoe ns,smtt   h ne etie h,hregtrs nigtike,aoaenns lng
```

train more

```
"Tmont thithey" fomesscerliund
Keushey. Thom here
sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome
coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."
```

train more

```
Aftair fall unsuch that the hall for Prince Velzonski's that me of
her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort
how, and Gogition is so overelical and ofter.
```

train more

```
"Why do what that day," replied Natasha, and wishing to himself the fact the
princess, Princess Mary was easier, fed in had oftened him.
Pierre aking his soul came to the packs and drove up his father-in-law women.
```

```
PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:
They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:
Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I'll have the heart of the wars.

Clown:
Come, sir, I will make did behold your worship.

VIOLA:
I'll drink it.
```

```
VIOLA:
Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.
```

Image Captioning

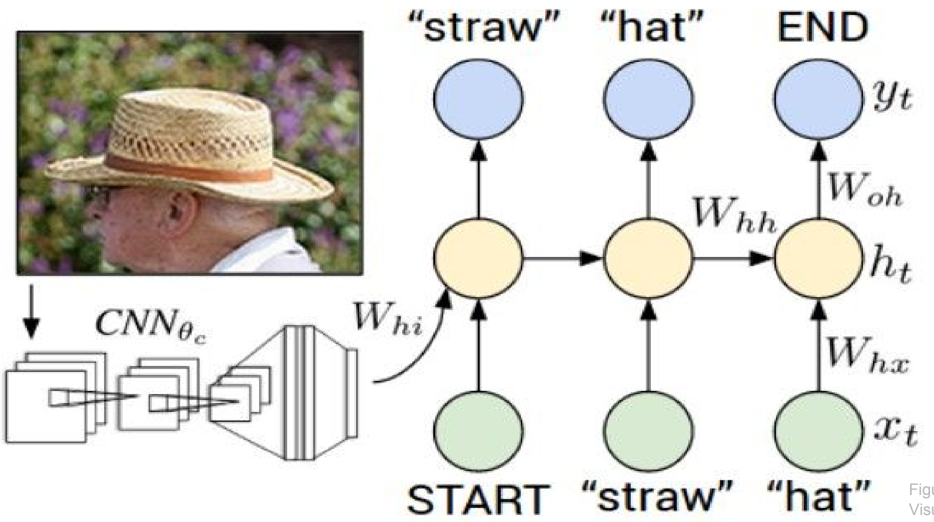


Figure from Karpathy et a, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015; figure copyright IEEE, 2015.
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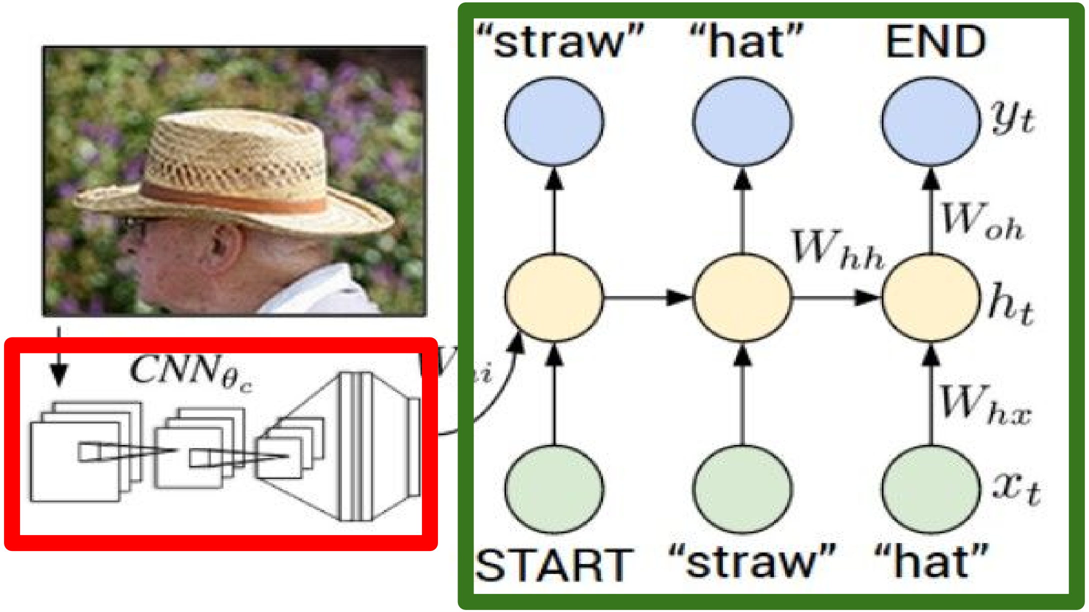
Explain Images with Multimodal Recurrent Neural Networks, Mao et al.
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei
Show and Tell: A Neural Image Caption Generator, Vinyals et al.
Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.
Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

Recurrent Neural Network



Convolutional Neural Network

test image

image

conv-64
conv-64
maxpool

conv-128
conv-128
maxpool

conv-256
conv-256
maxpool

conv-512
conv-512
maxpool

conv-512
conv-512
maxpool

FC-4096
FC-4096
FC-1000
softmax

test image

image

conv-64
conv-64
maxpool

conv-128
conv-128
maxpool

conv-256
conv-256
maxpool

conv-512
conv-512
maxpool

conv-512
conv-512
maxpool

FC-4096
FC-4096
~~FC-1000~~
~~softmax~~

test image

image

conv-64
conv-64
maxpool

conv-128
conv-128
maxpool

conv-256
conv-256
maxpool

conv-512
conv-512
maxpool

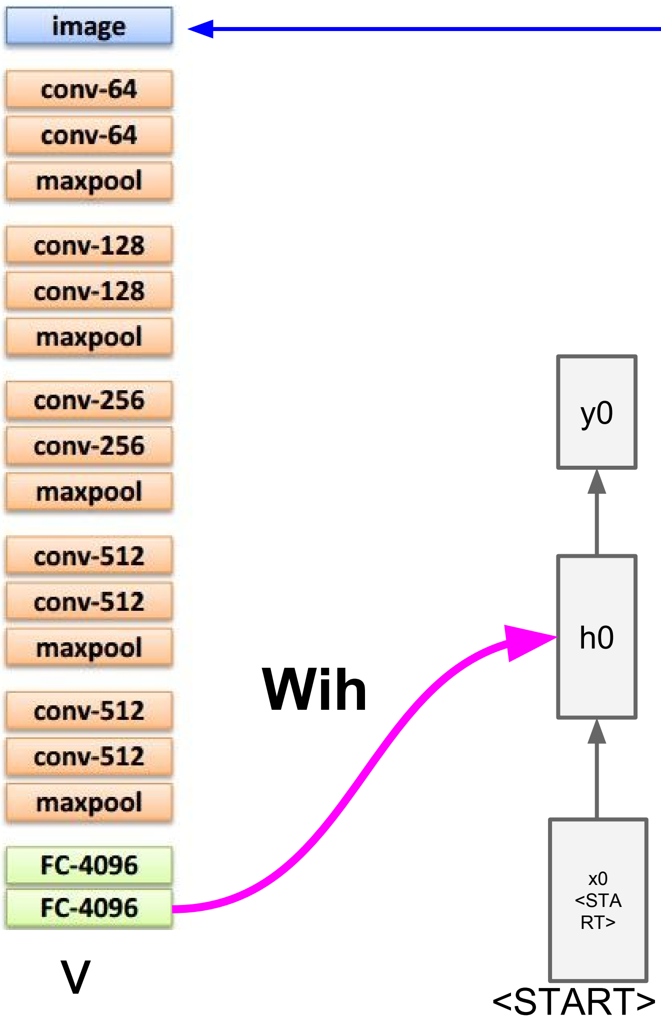
conv-512
conv-512
maxpool

FC-4096
FC-4096

test image

x0
<START>

<START>



test image

before:

$$h = \tanh(Wxh * x + Whh * h)$$

now:

$$h = \tanh(Wxh * x + Whh * h + Wih * v)$$

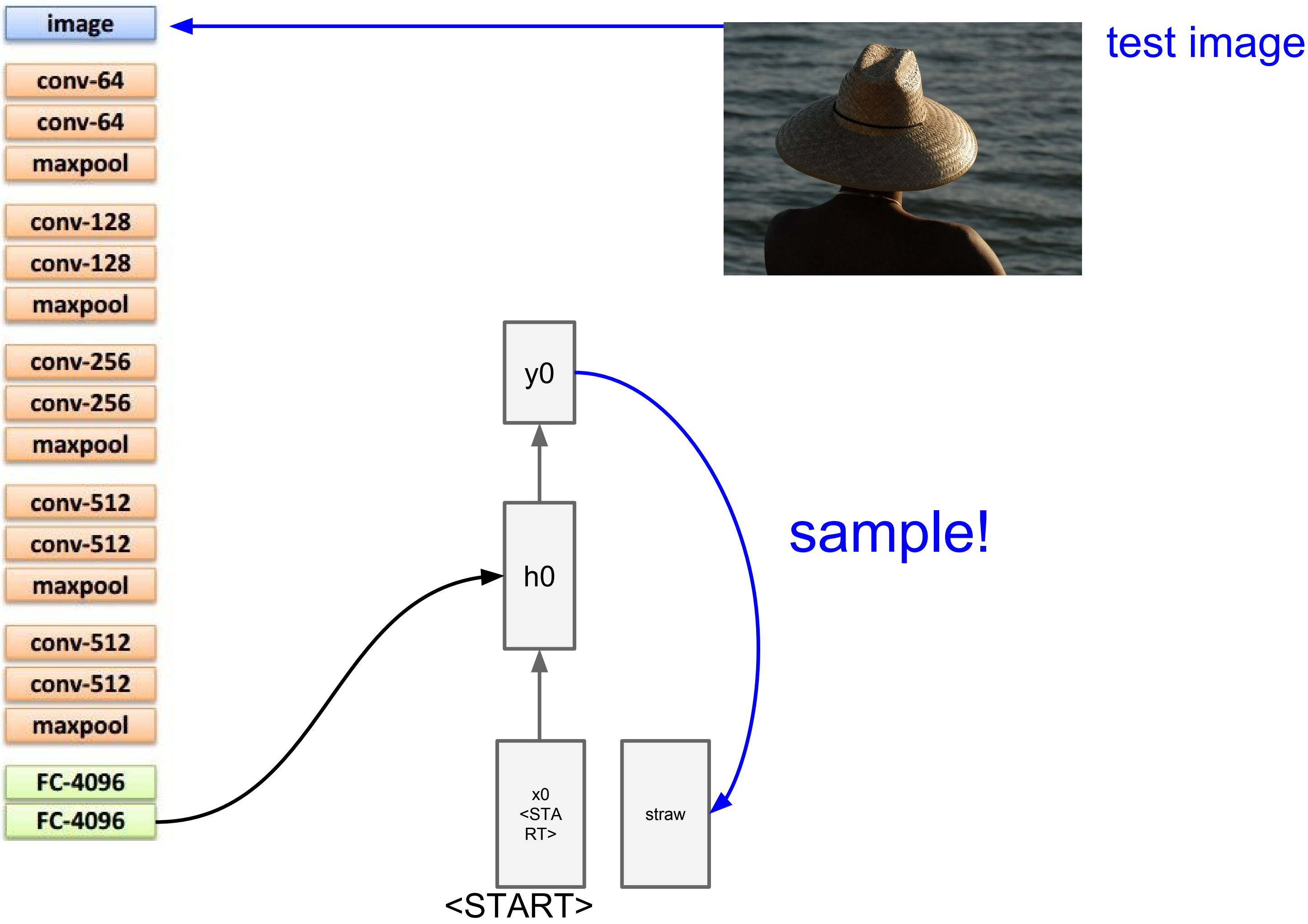


image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
h0
sample!
x0
<STA
RT>
straw
<START>

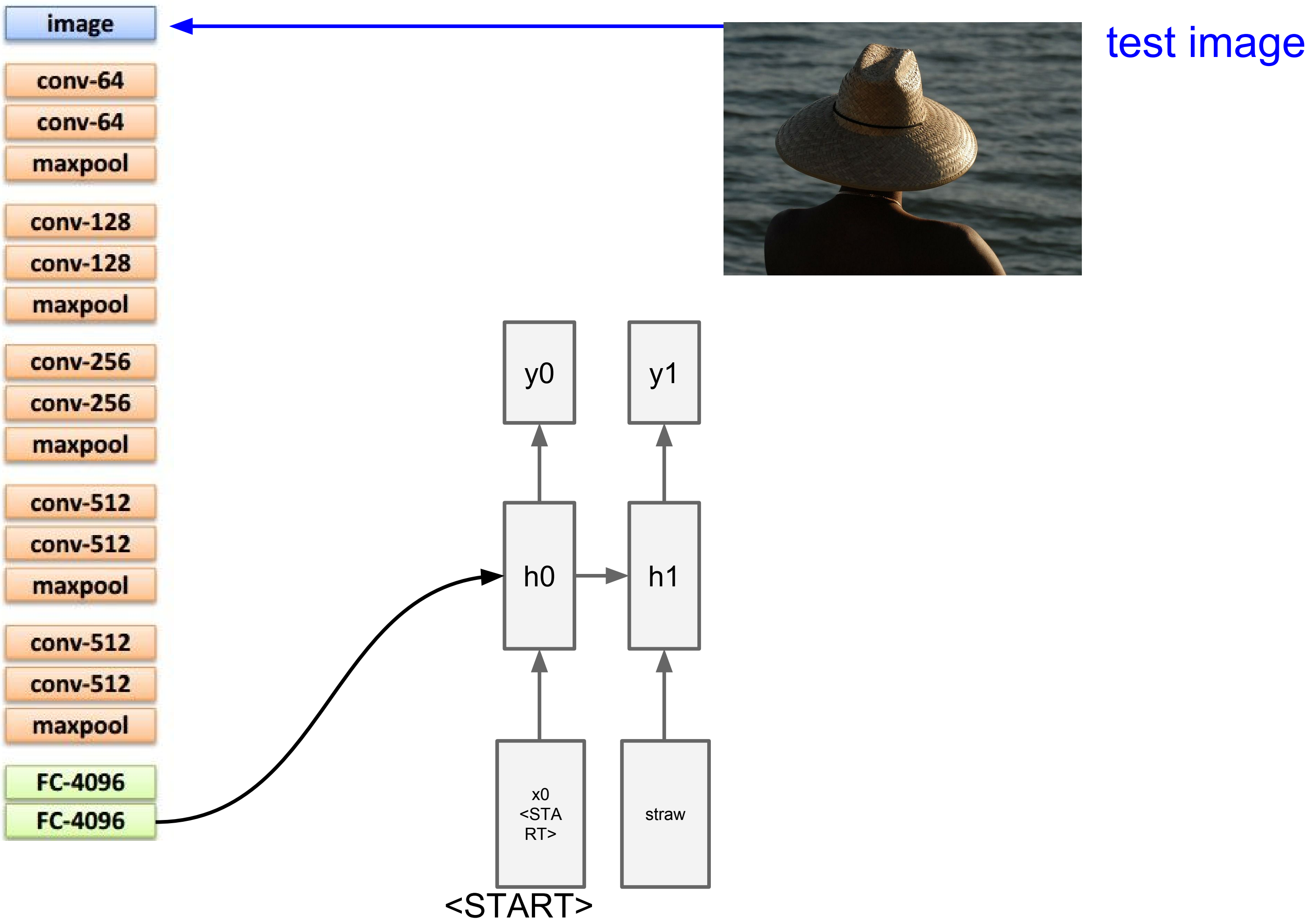


image
conv-64
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maxpool
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conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
y1
h0
h1
x0 <STA RT>
straw
<START>

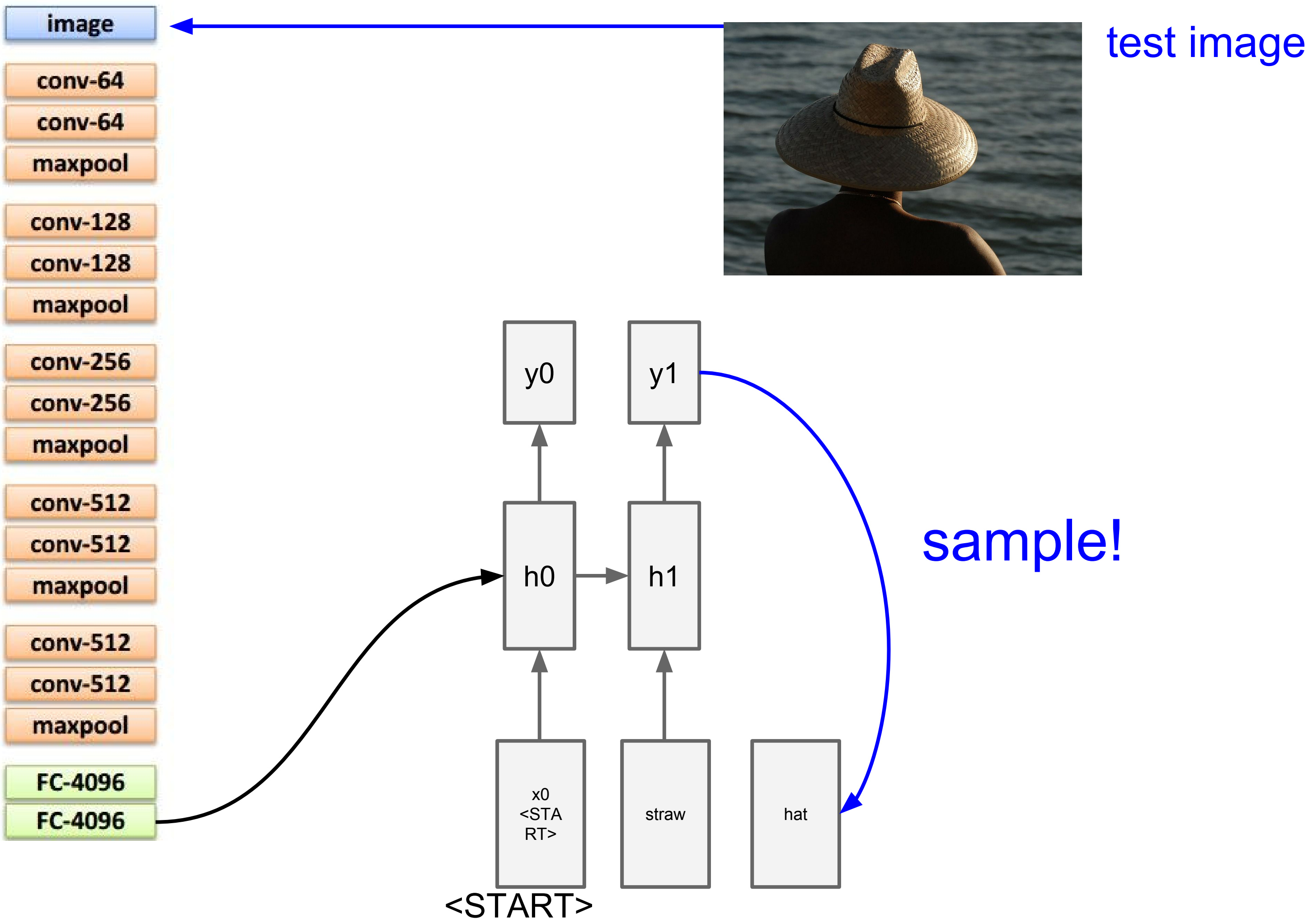
image
conv-64
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maxpool
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conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
y1
h0
h1
sample!
x0
<STA
RT>
straw
hat
<START>

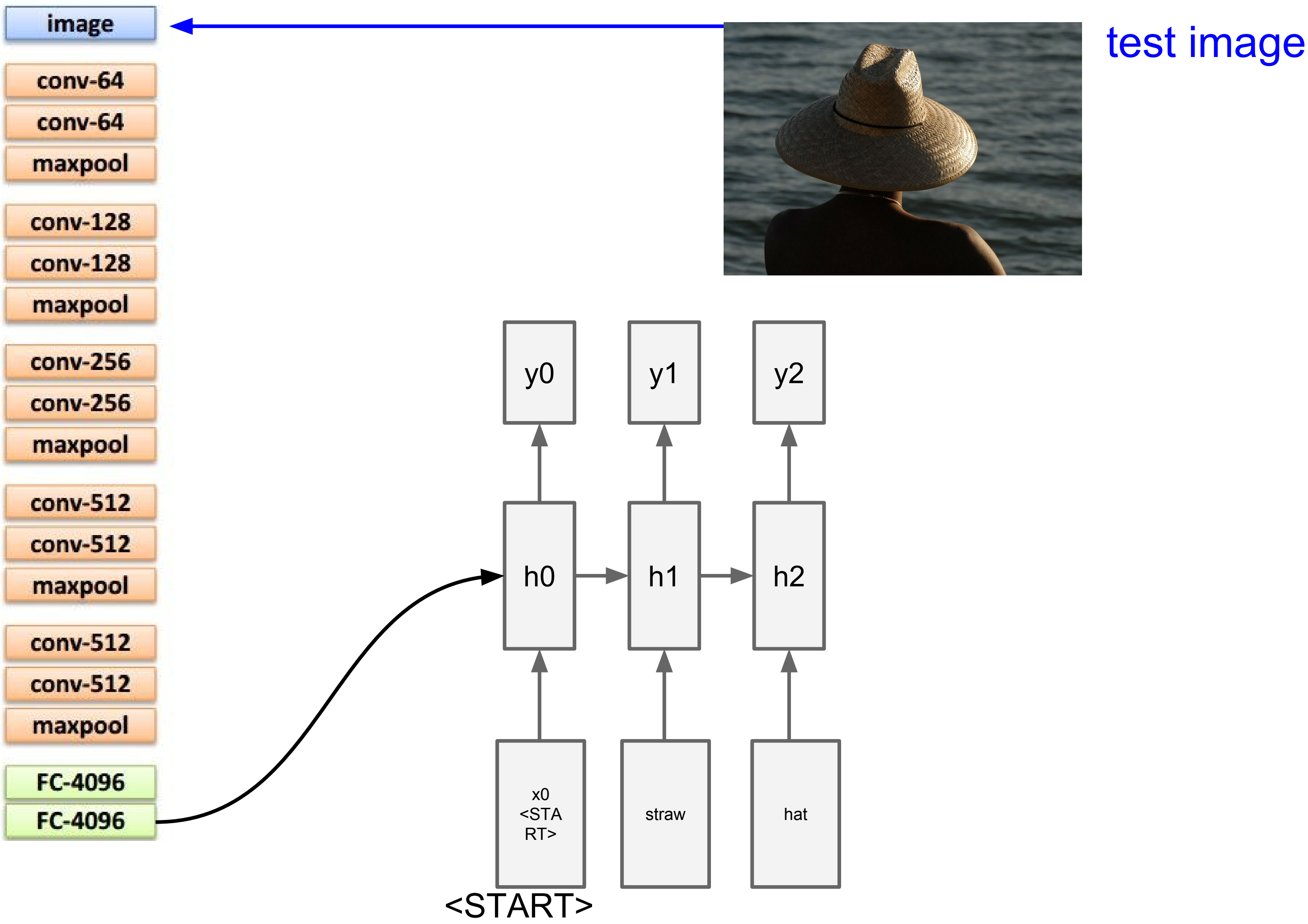
image
conv-64
conv-64
maxpool
conv-128
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conv-256
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maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
y1
y2
h0
h1
h2
x0 <STA RT>
straw
hat
<START>

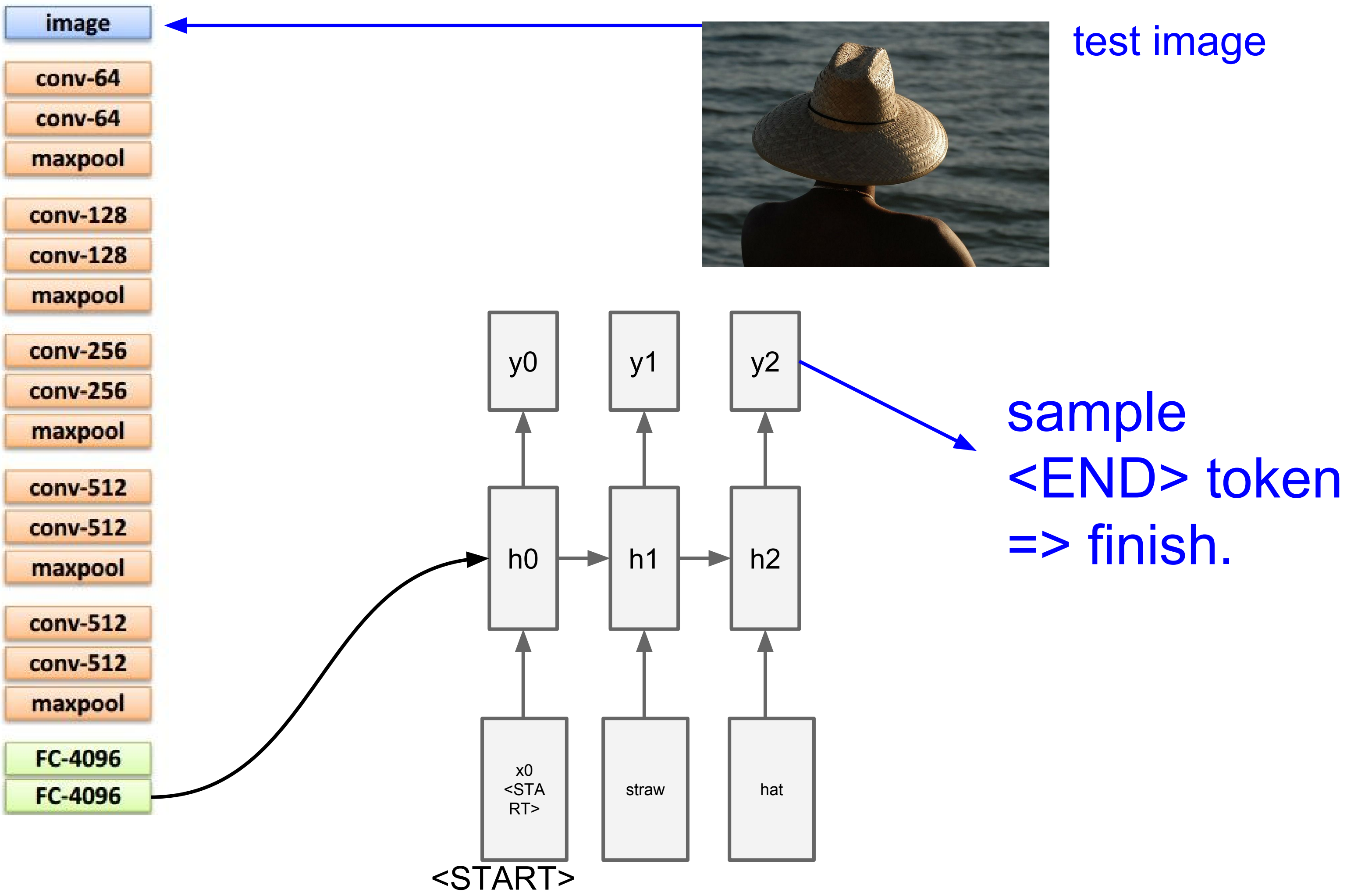
image
conv-64
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maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
test image
y0
y1
y2
h0
h1
h2
x0 <STA RT>
straw
hat
<START>
sample
<END> token
=> finish.

Image Captioning: Example Results

A cat sitting on a suitcase on the floor

A cat is sitting on a tree branch

A dog is running in the grass with a frisbee

A white teddy bear sitting in the grass

Two people walking on the beach with surfboards

A tennis player in action on the court

Two giraffes standing in a grassy field

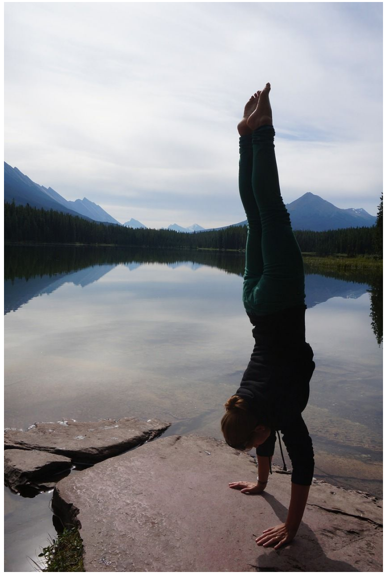
A man riding a dirt bike on a dirt track

Image Captioning: Failure Cases

Captions generated using neuraltalk2
All images are CC0 Public domain: fur coat, handstand, spider web, baseball

A woman is holding a cat in her hand

A person holding a computer mouse on a desk



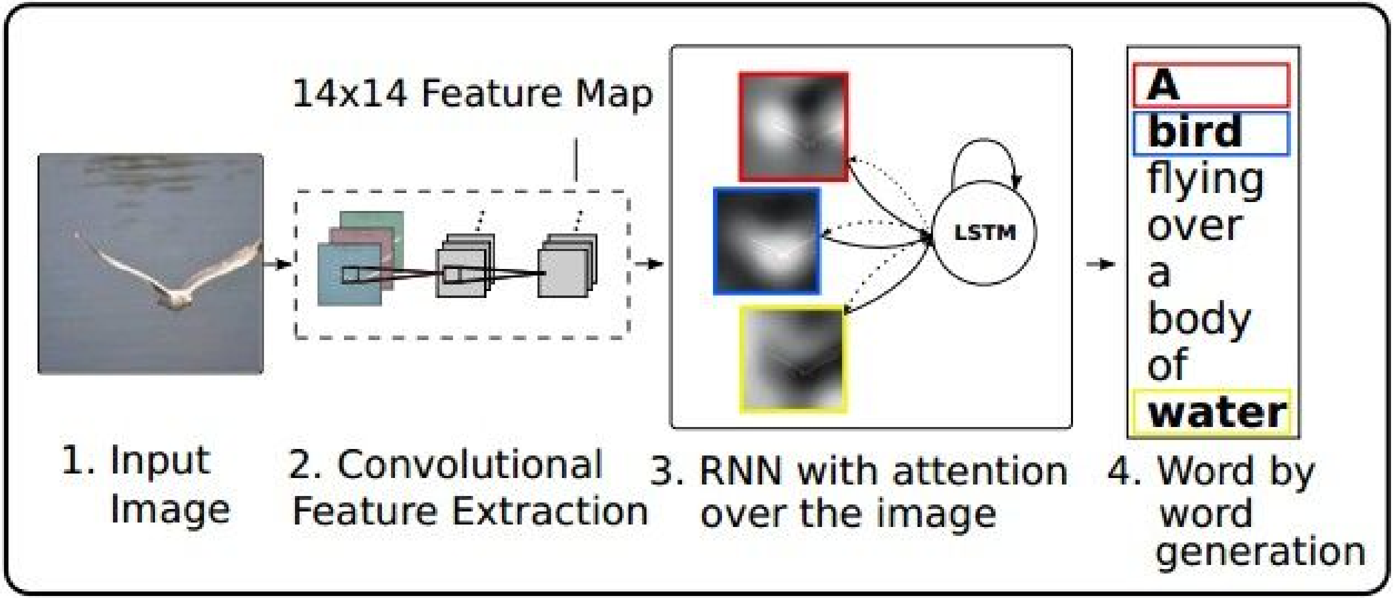
A woman standing on a beach holding a surfboard

A bird is perched on a tree branch

A man in a baseball uniform throwing a ball

Image Captioning with Attention

RNN focuses its attention at a different spatial location when generating each word



Xu et al, “Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention”, ICML 2015

Image Captioning with Attention

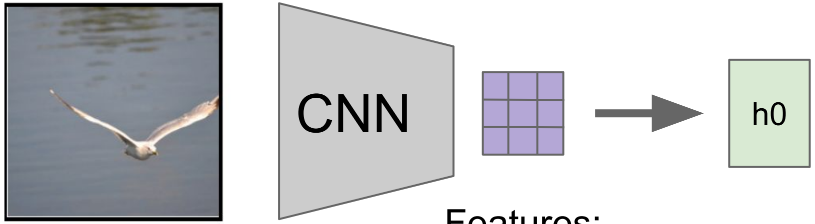
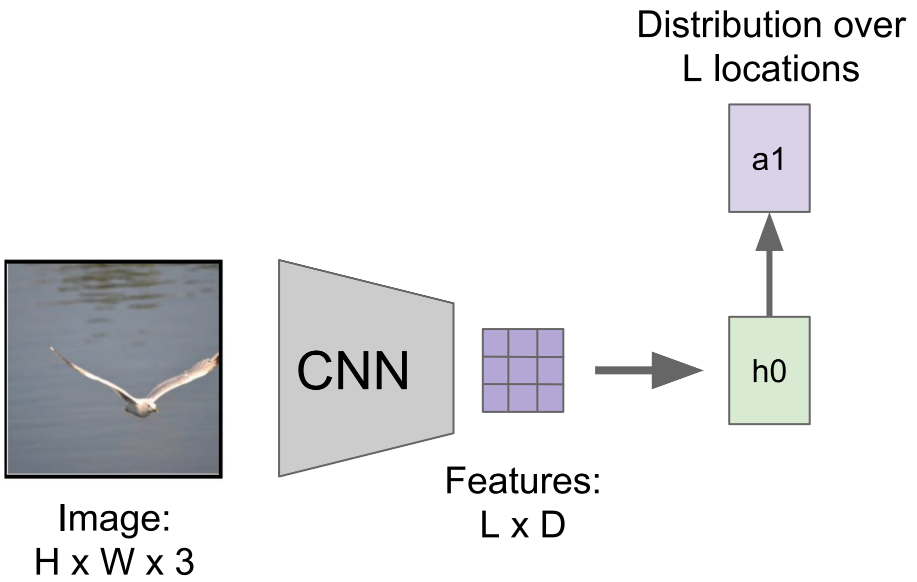


Image:
H x W x 3

Features:
L x D

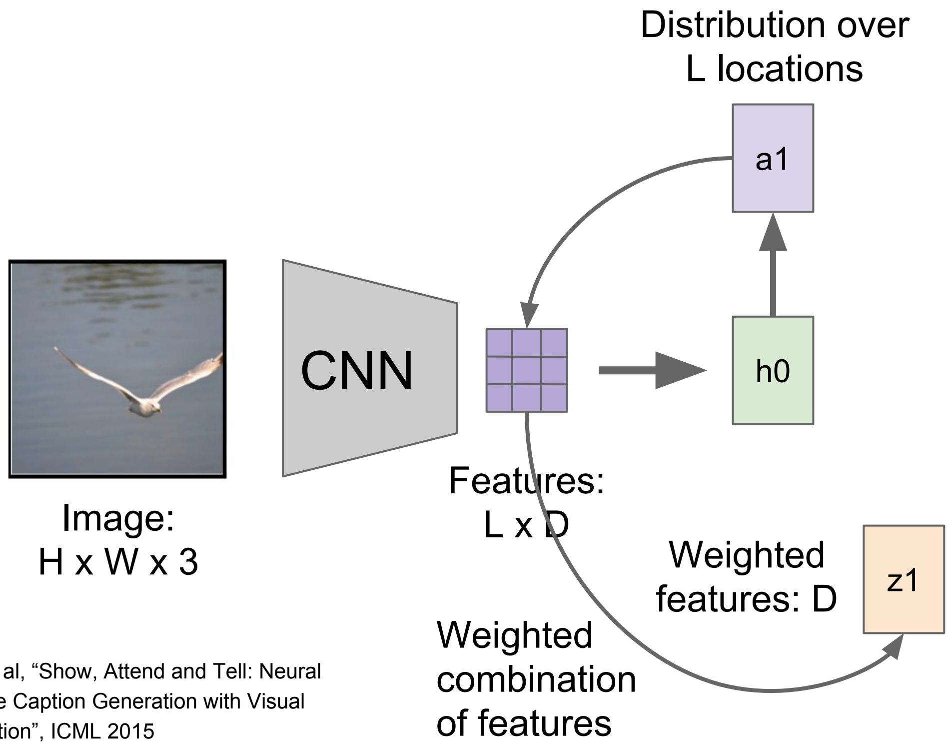
Xu et al, “Show, Attend and Tell: Neural Image Caption Generation with Visual Attention”, ICML 2015

Image Captioning with Attention



Xu et al, “Show, Attend and Tell: Neural Image Caption Generation with Visual Attention”, ICML 2015

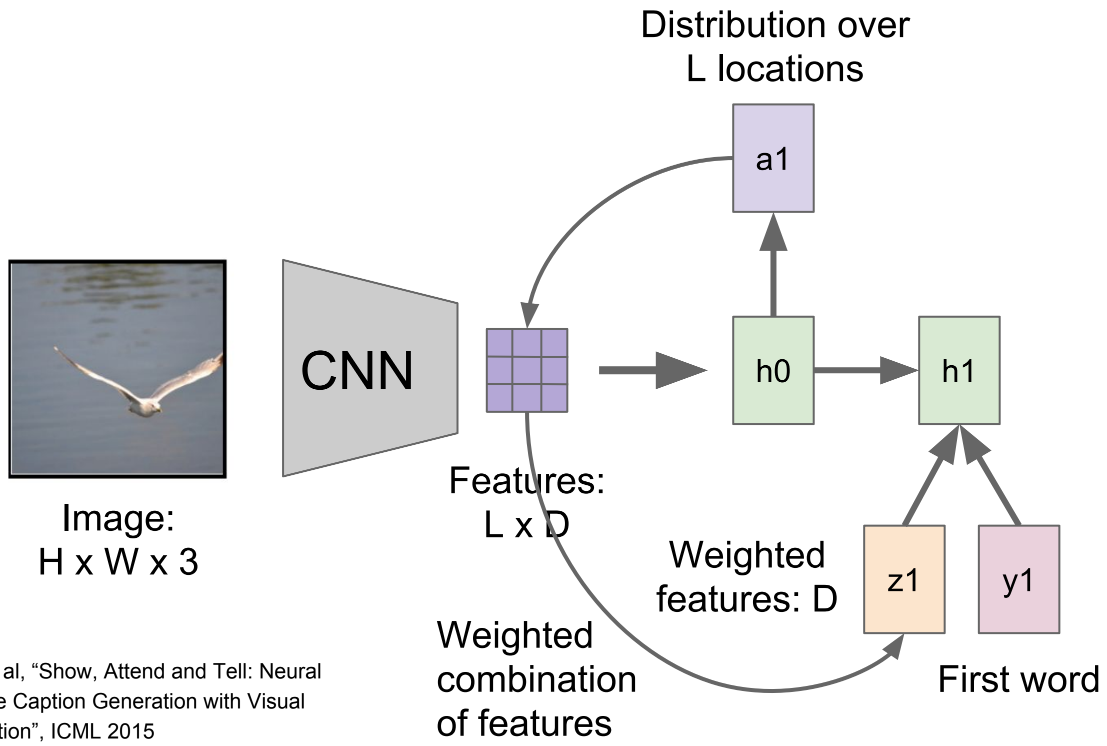
Image Captioning with Attention



$$z = \sum_{i=1}^{L} p_i v_i$$

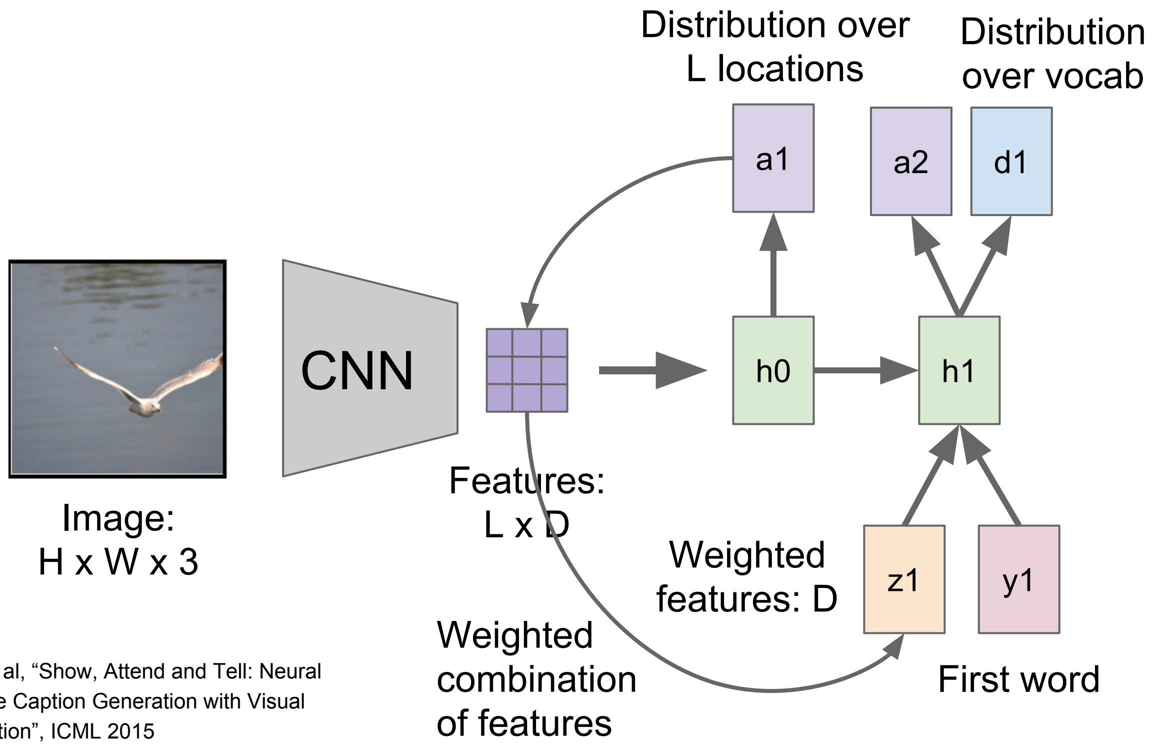
Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



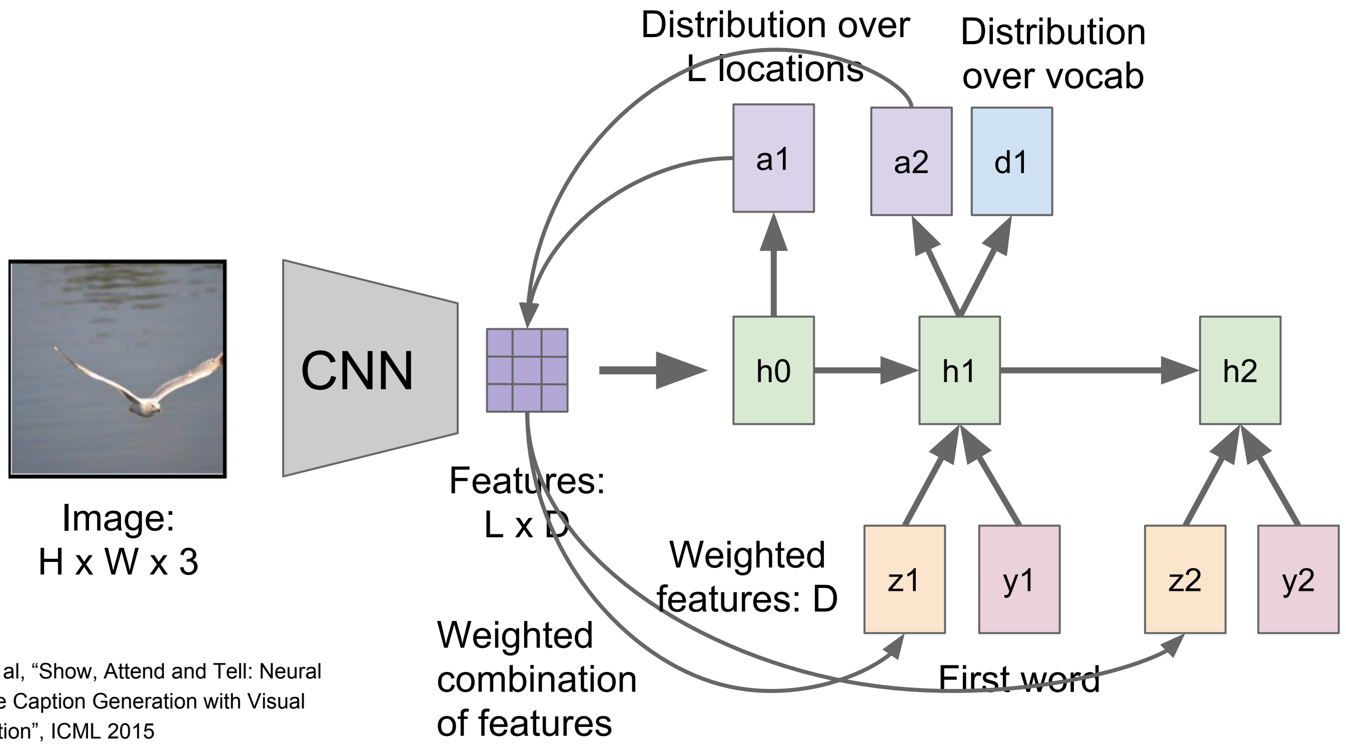
Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



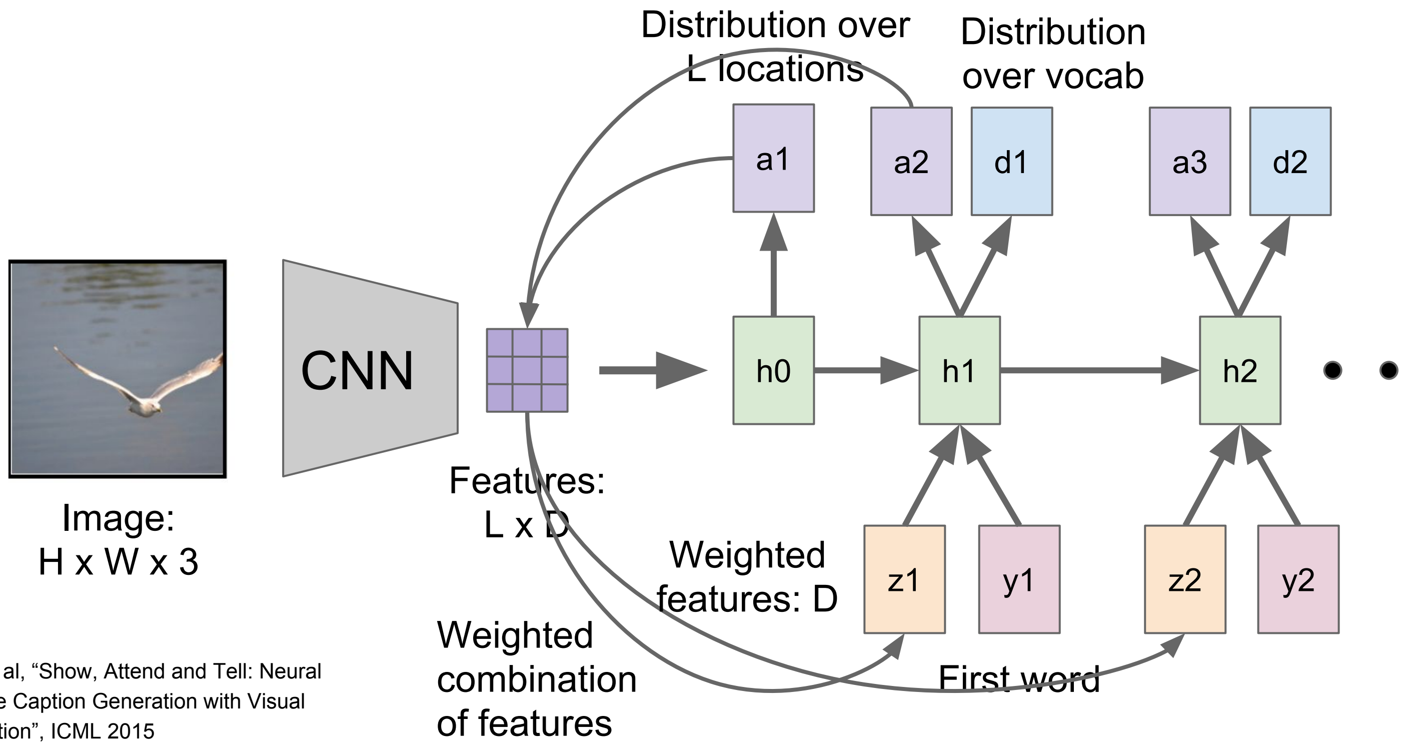
Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



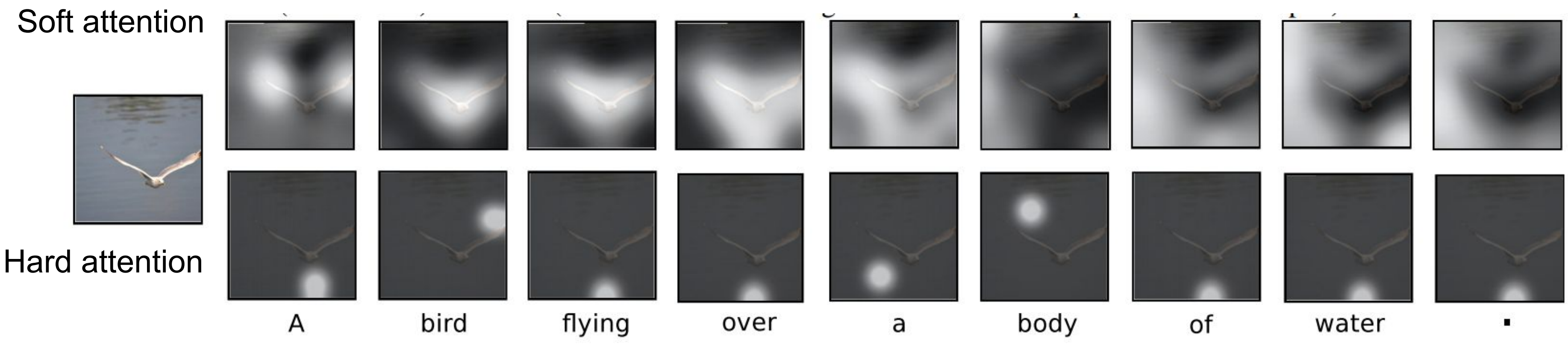
Xu et al, "Show, Attend and Tell: Neural Image Caption Generation with Visual Attention", ICML 2015

Image Captioning with Attention



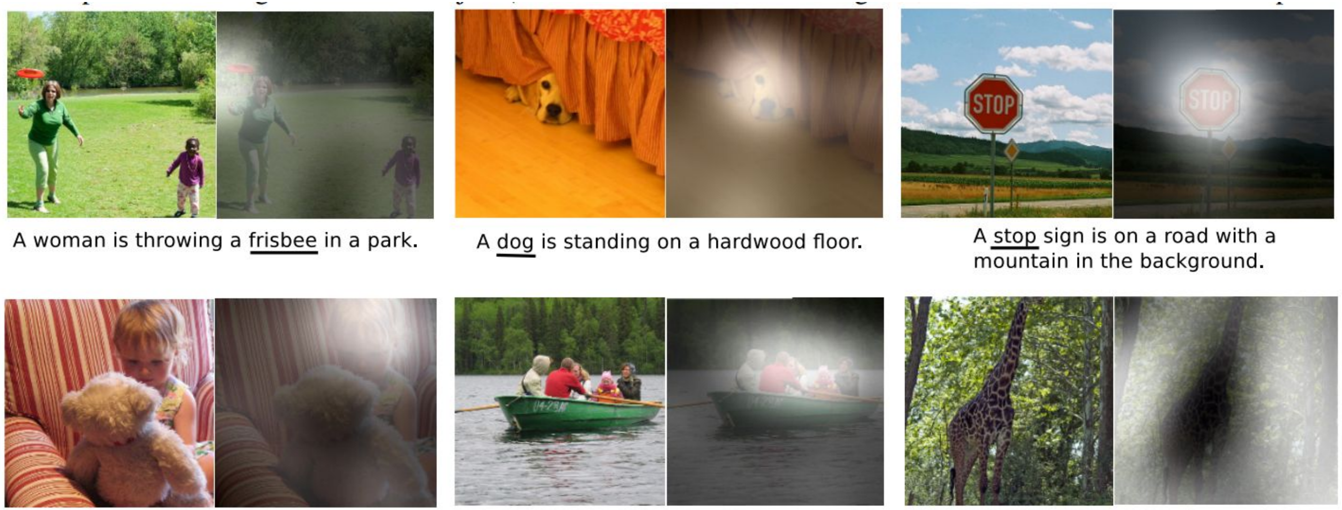
Xu et al, “Show, Attend and Tell: Neural Image Caption Generation with Visual Attention”, ICML 2015

Image Captioning with Attention



Xu et al, “Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention”, ICML 2015

Image Captioning with Attention



A woman is throwing a frisbee in a park.

A dog is standing on a hardwood floor.

A stop sign is on a road with a mountain in the background.

A little girl sitting on a bed with a teddy bear.

A group of people sitting on a boat in the water.

A giraffe standing in a forest with trees in the background.

Xu et al, “Show, Attend, and Tell: Neural Image Caption Generation with Visual Attention”, ICML 2015

Visual Question Answering

Q: What endangered animal is featured on the truck?

A: **A bald eagle.**
A: A sparrow.
A: A humming bird.
A: A raven.

Q: Where will the driver go if turning right?

A: **Onto 24 ¾ Rd.**
A: Onto 25 ¾ Rd.
A: Onto 23 ¾ Rd.
A: Onto Main Street.

Q: When was the picture taken?

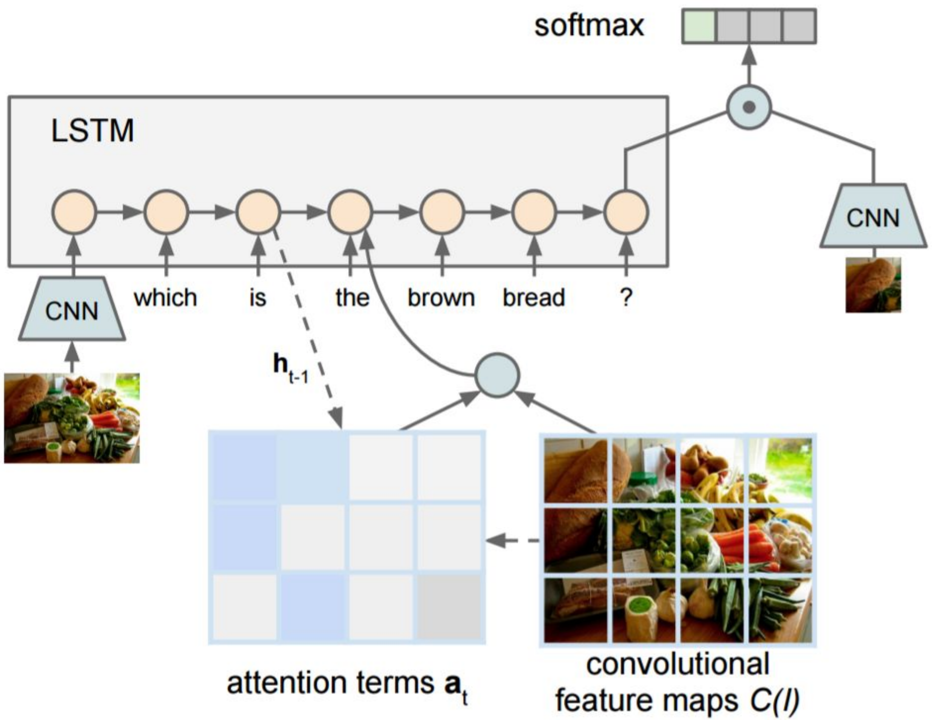
A: **During a wedding.**
A: During a bar mitzvah.
A: During a funeral.
A: During a Sunday church service

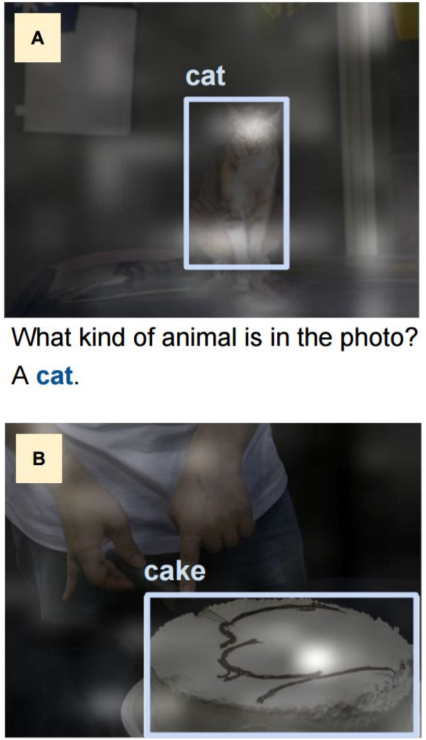
Q: Who is under the umbrella?

A: **Two women.**
A: A child.
A: An old man.
A: A husband and a wife.

Agrawal et al, “VQA: Visual Question Answering”, ICCV 2015
Zhu et al, “Visual 7W: Grounded Question Answering in Images”, CVPR 2016

Visual Question Answering: RNNs with Attention





What kind of animal is in the photo?
A **cat**.

Why is the person holding a knife?
To cut the **cake** with.

Zhu et al, "Visual 7W: Grounded Question Answering in Images", CVPR 2016

Multilayer RNNs

$$h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$h \in \mathbb{R}^n$ $\quad W^l \ [n \times 2n]$

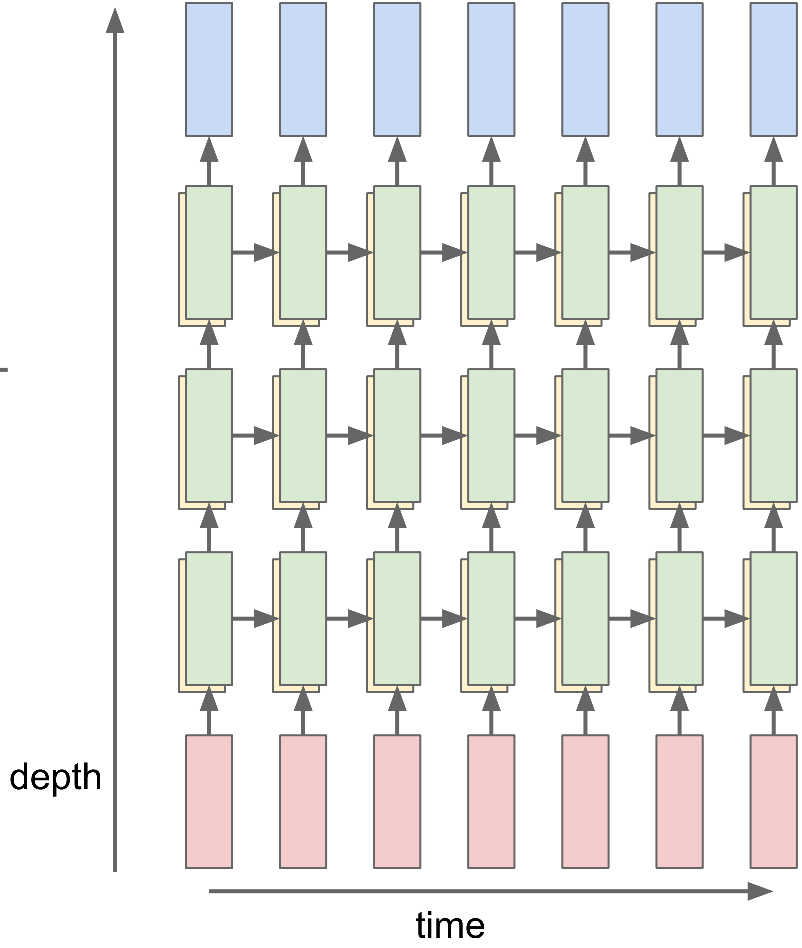
LSTM:

$W^l \ [4n \times 2n]$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

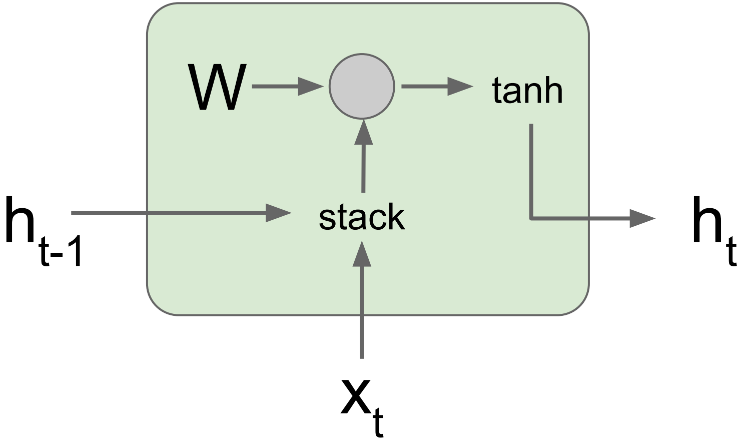
$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

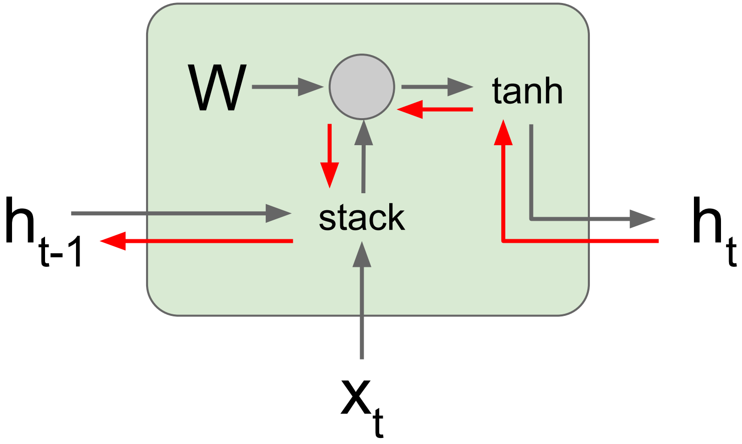


$$
\begin{aligned}
h_t &= \tanh(W_{hh}h_{t-1} + W_{xh}x_t) \\
&= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right) \\
&= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)
\end{aligned}
$$

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

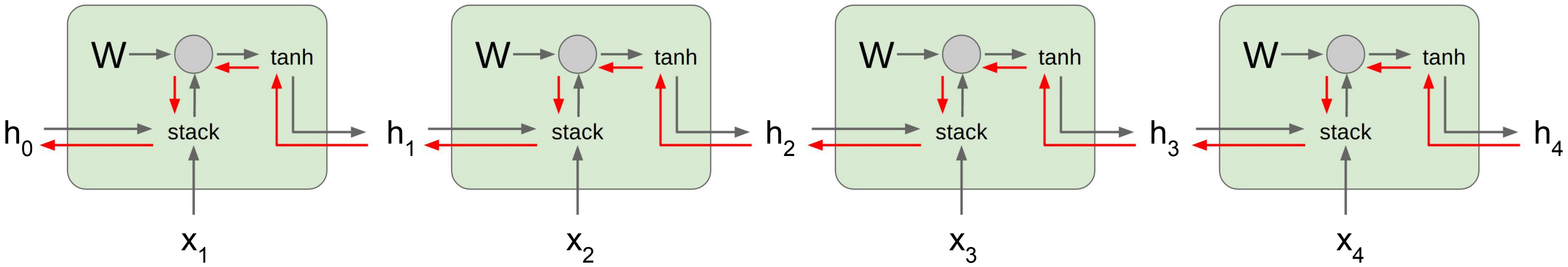
Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh}^T)



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$
$$= \tanh\left(\begin{pmatrix} W_{hh} & W_{hx} \end{pmatrix} \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$
$$= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

Vanilla RNN Gradient Flow

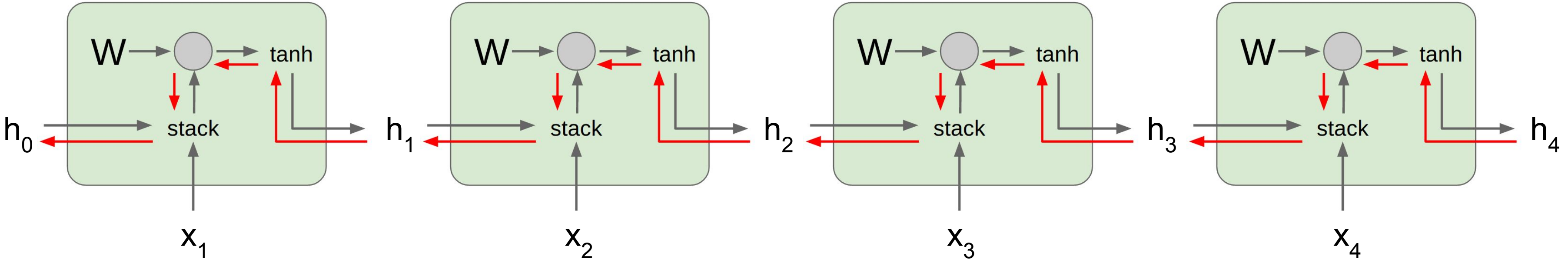
Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Vanilla RNN Gradient Flow

Bengio et al, “Learning long-term dependencies with gradient descent is difficult”, IEEE Transactions on Neural Networks, 1994
Pascanu et al, “On the difficulty of training recurrent neural networks”, ICML 2013



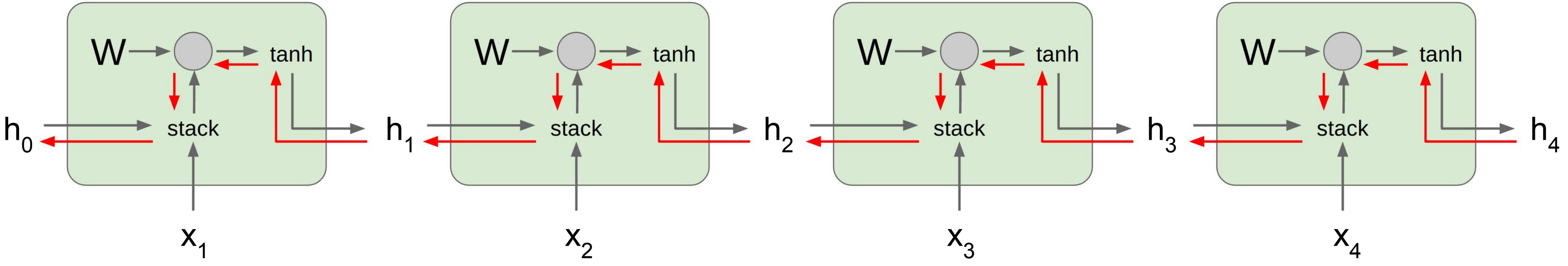
Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W
(and repeated tanh)

Largest singular value > 1:
Exploding gradients

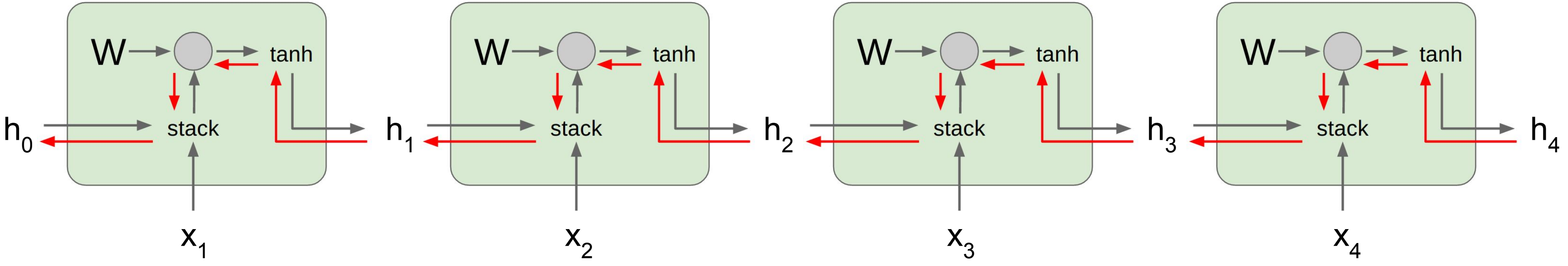
Gradient clipping: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

→ Change RNN architecture

Long Short Term Memory (LSTM)

Vanilla RNN

$$\boxed{h_t = \tanh\left(W\begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}\right)}$$

LSTM

$$\begin{pmatrix}i\\f\\o\\g\end{pmatrix} = \begin{pmatrix}\sigma\\\sigma\\\sigma\\\tanh\end{pmatrix} W \begin{pmatrix}h_{t-1}\\x_t\end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

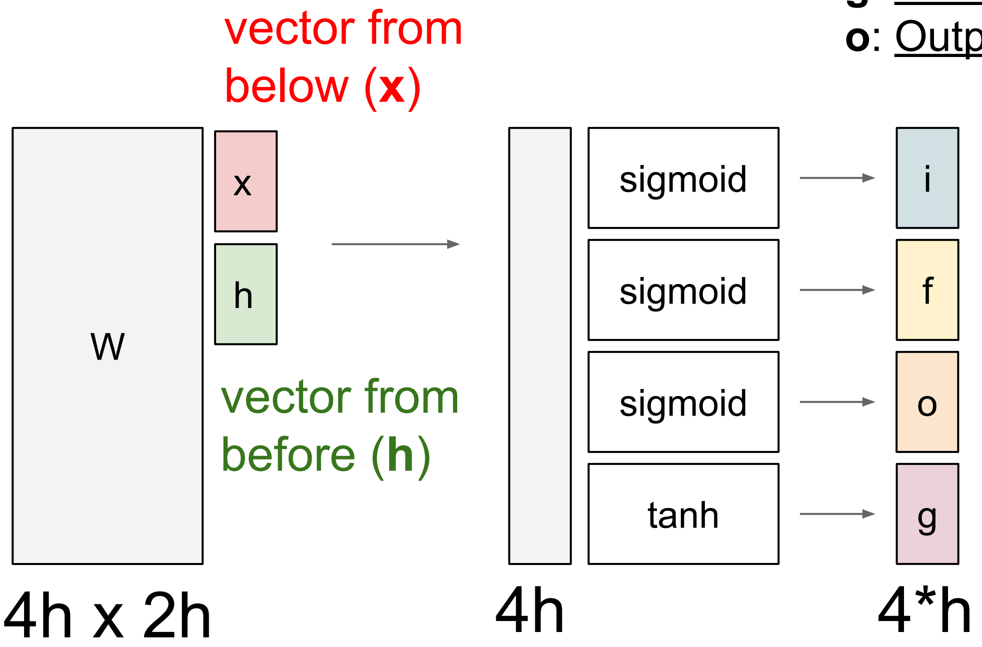
$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, “Long Short Term Memory”, Neural Computation 1997

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]

f: Forget gate, Whether to erase cell
i: Input gate, whether to write to cell
g: Gate gate (?), How much to write to cell
o: Output gate, How much to reveal cell



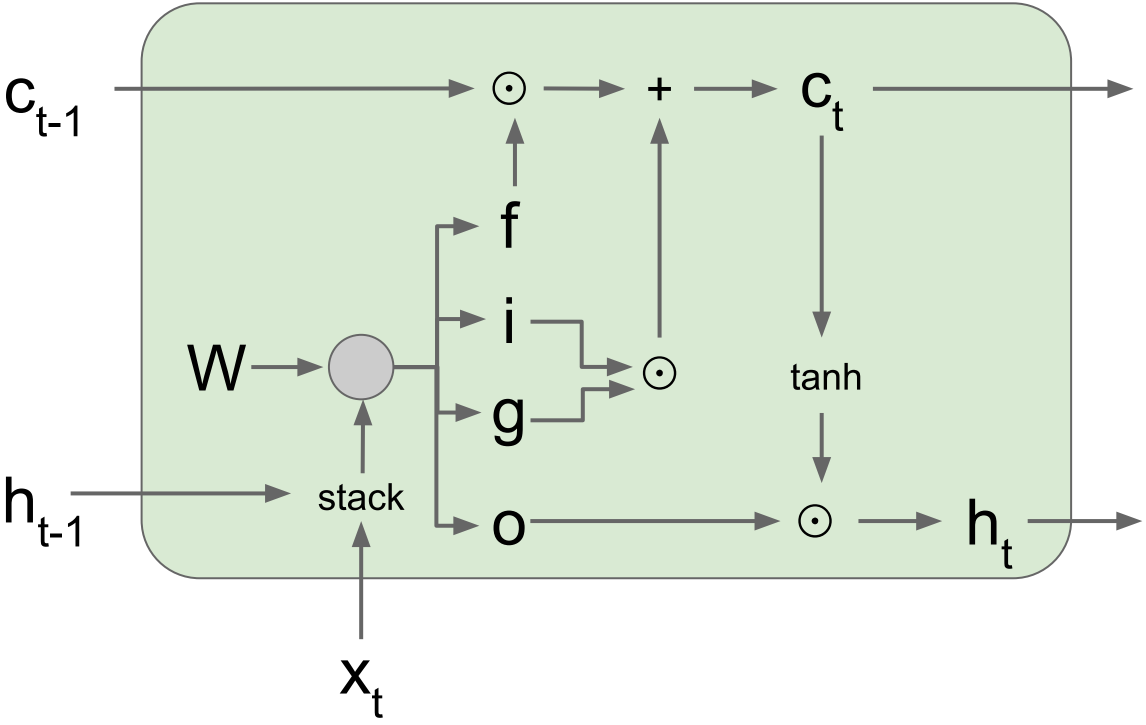
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



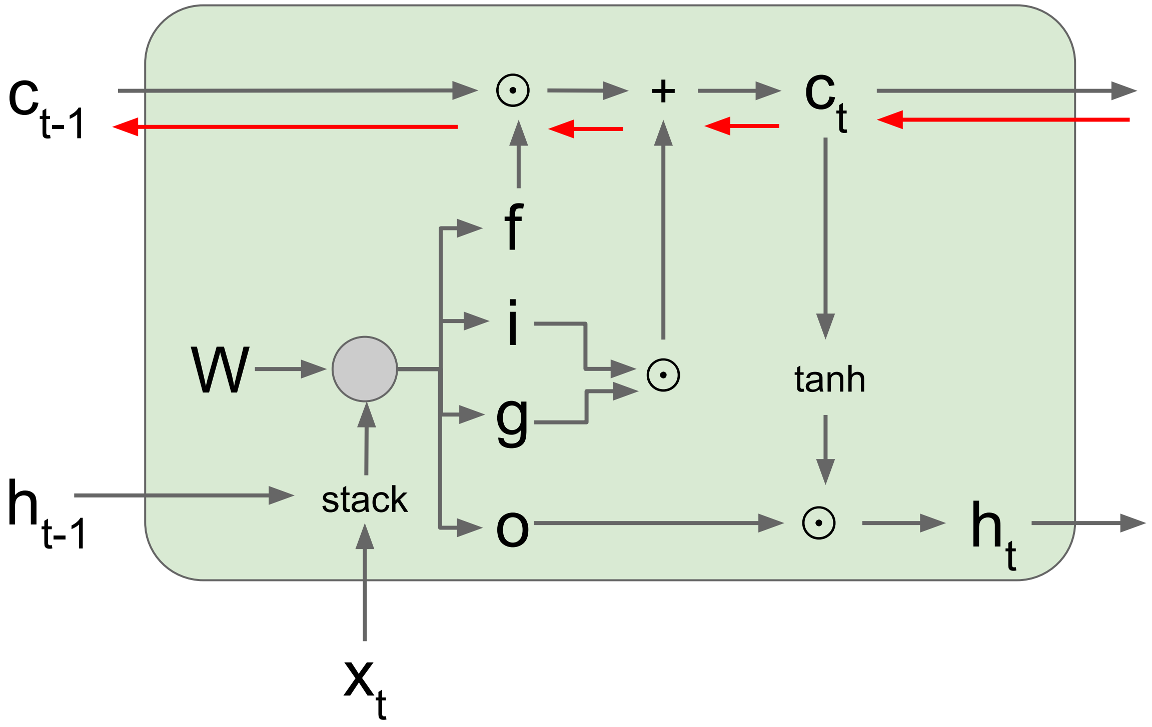
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f, no matrix multiply by W

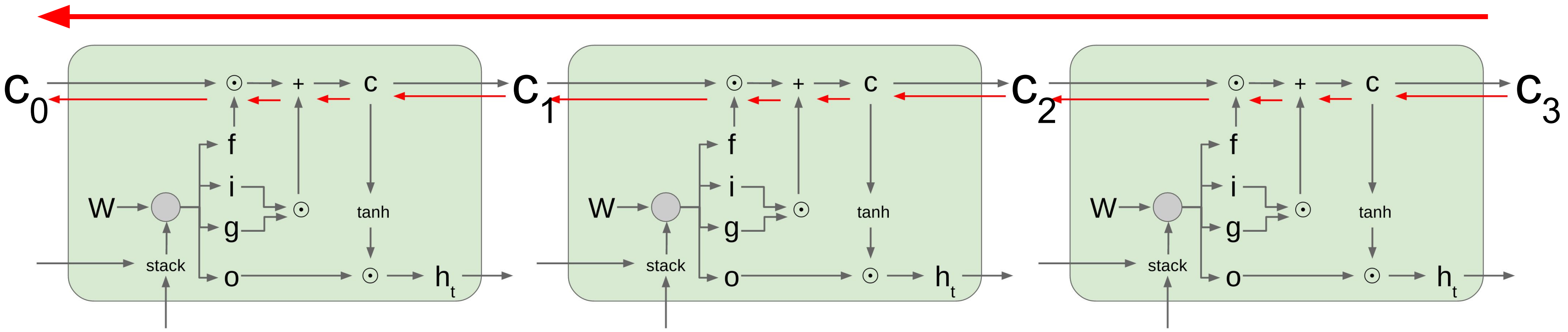
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow
[Hochreiter et al., 1997]

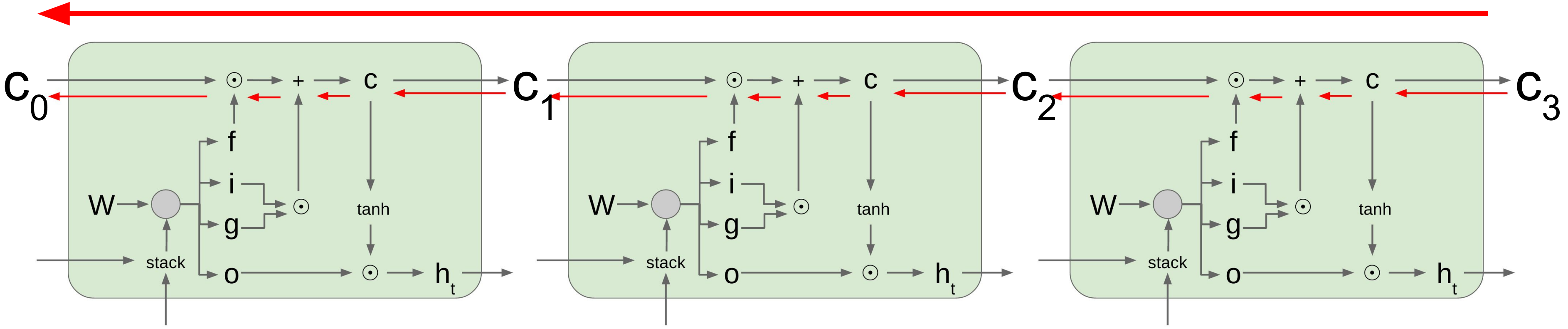
Uninterrupted gradient flow!



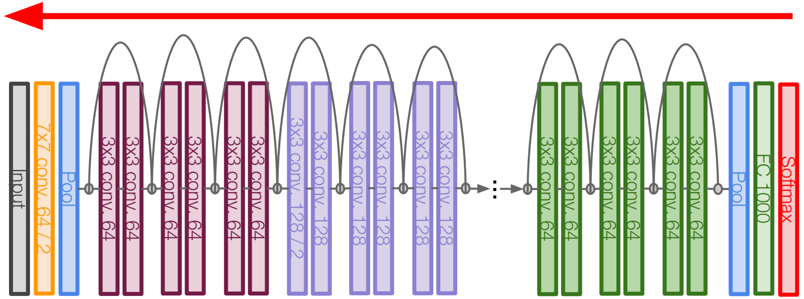
Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



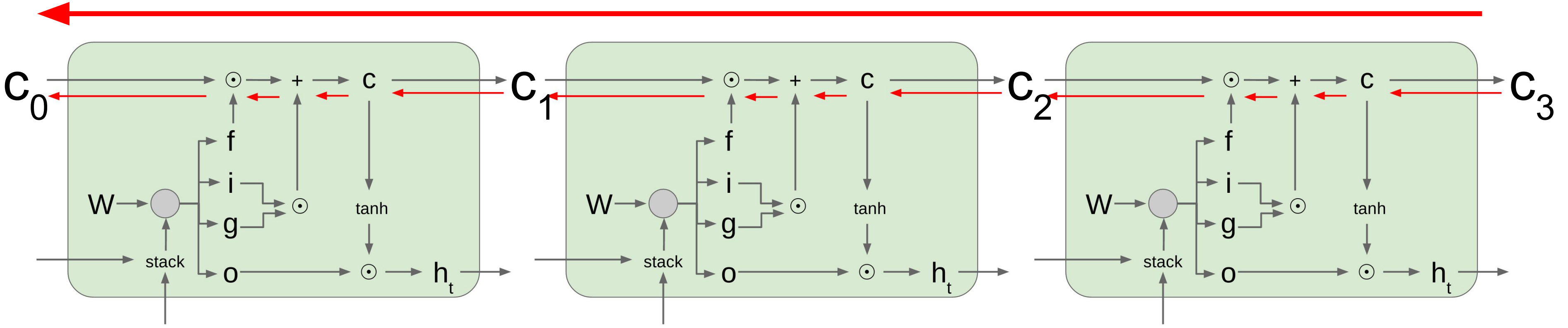
Similar to ResNet!



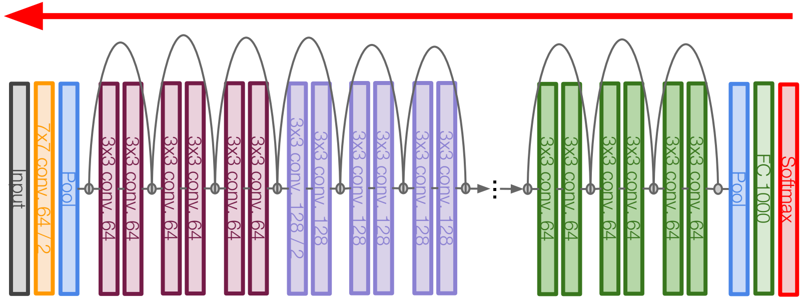
Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



Similar to ResNet!



In between:

Highway Networks

$$g = T(x, W_T)$$

$$y = g \odot H(x, W_H) + (1 - g) \odot x$$

Srivastava et al, “Highway Networks”, ICML DL Workshop 2015

Other RNN Variants

[*An Empirical Exploration of Recurrent Network Architectures*, Jozefowicz et al., 2015]

GRU [*Learning phrase representations using rnn encoder-decoder for statistical machine translation*, Cho et al. 2014]

$$
\begin{aligned}
r_t &= \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r) \\
z_t &= \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z) \\
\tilde{h}_t &= \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h) \\
h_t &= z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t
\end{aligned}
$$

[*LSTM: A Search Space Odyssey*, Greff et al., 2015]

MUT1:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}}x_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}}x_t + W_{\mathrm{hr}}h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + \tanh(x_t) + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT2:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}}x_t + W_{\mathrm{hz}}h_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(x_t + W_{\mathrm{hr}}h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh}x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT3:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}}x_t + W_{\mathrm{hz}}\tanh(h_t) + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}}x_t + W_{\mathrm{hr}}h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh}x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$