Data Mining: Itemsets

Héctor Corrada Bravo

University of Maryland, College Park, USA

Fannie Mae: 2018-09-03

Data Mining

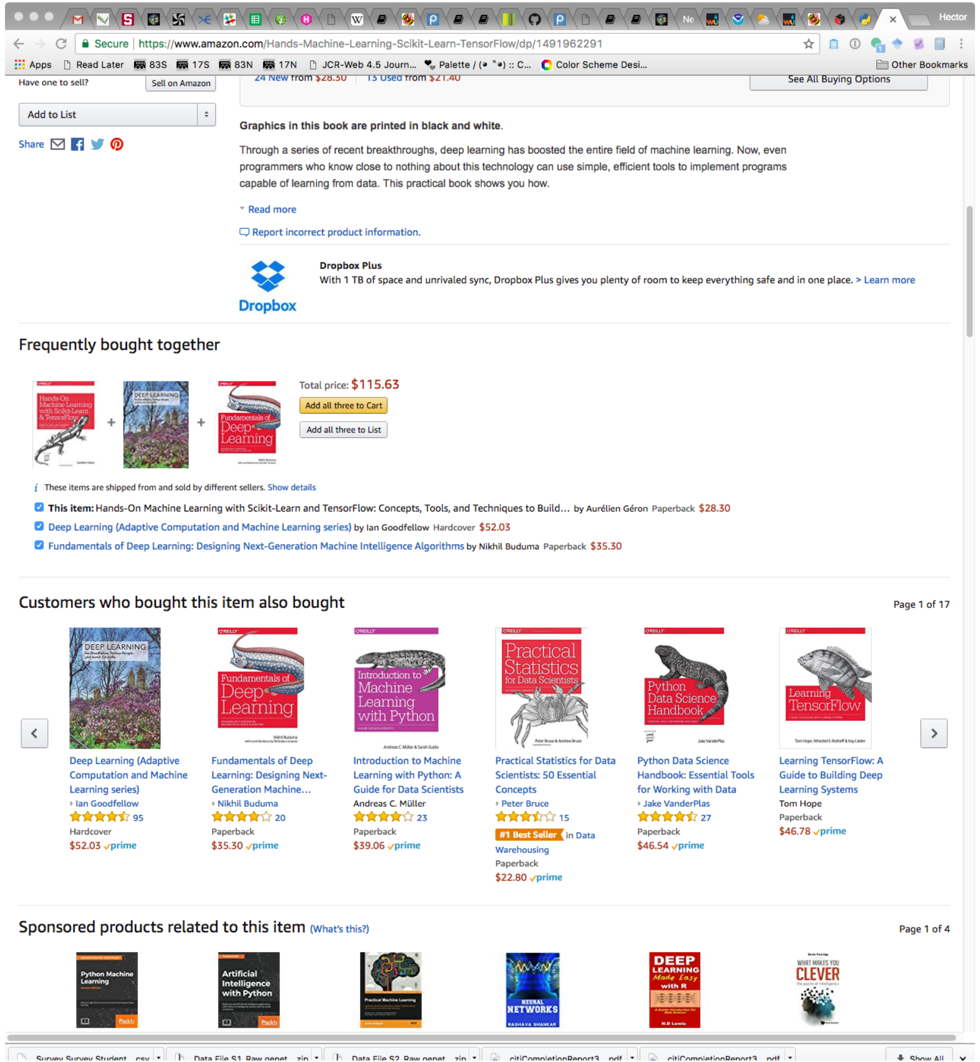
For today: the analysis of itemsets

- Similar itemsets
- Frequent itemsets

Similar Itemsets

Collaborative Filtering

- Items: customers
- Itemsets: customers that bought a specific book
- Similar itemsets: books purchased by same customers



Similar Itemsets

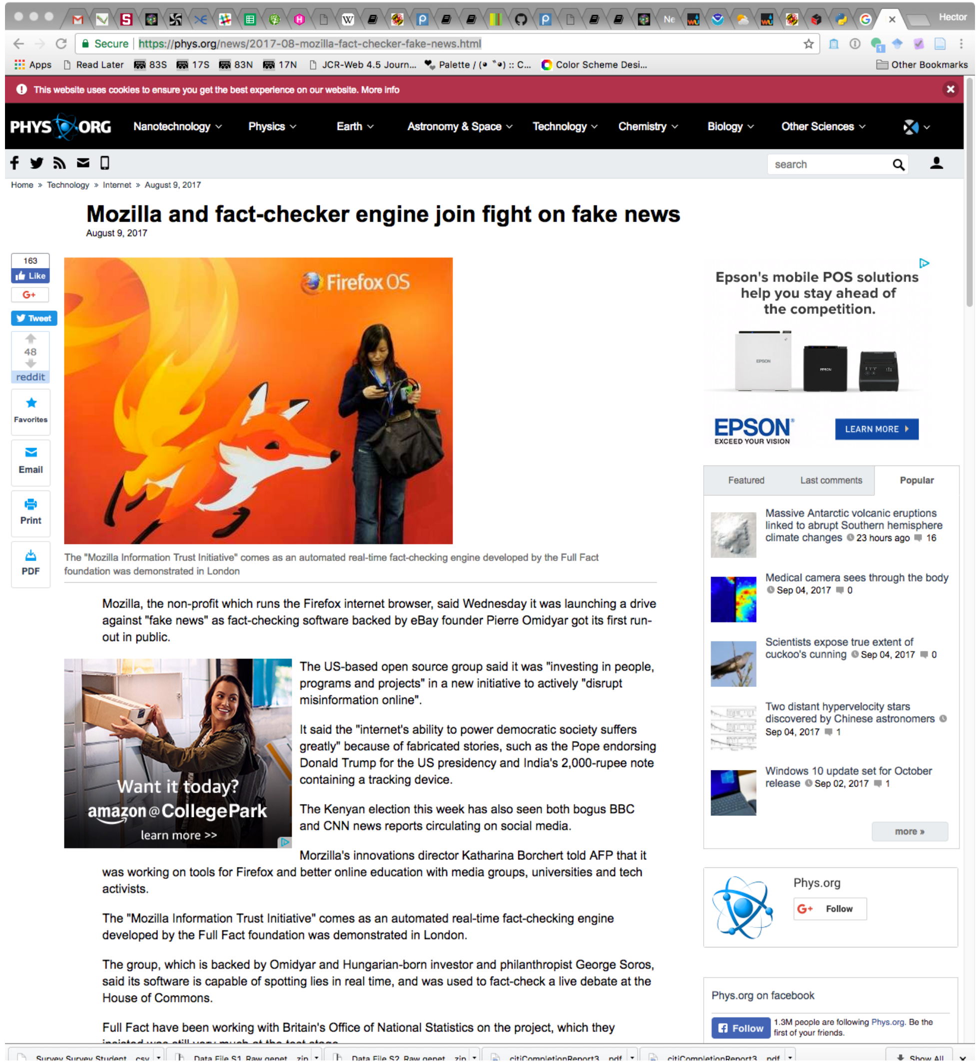
Similar Documents

- Items: words
- Itemsets: documents
- Similar itemsets: documents using many of the same words

Similar Itemsets

Similar News Articles

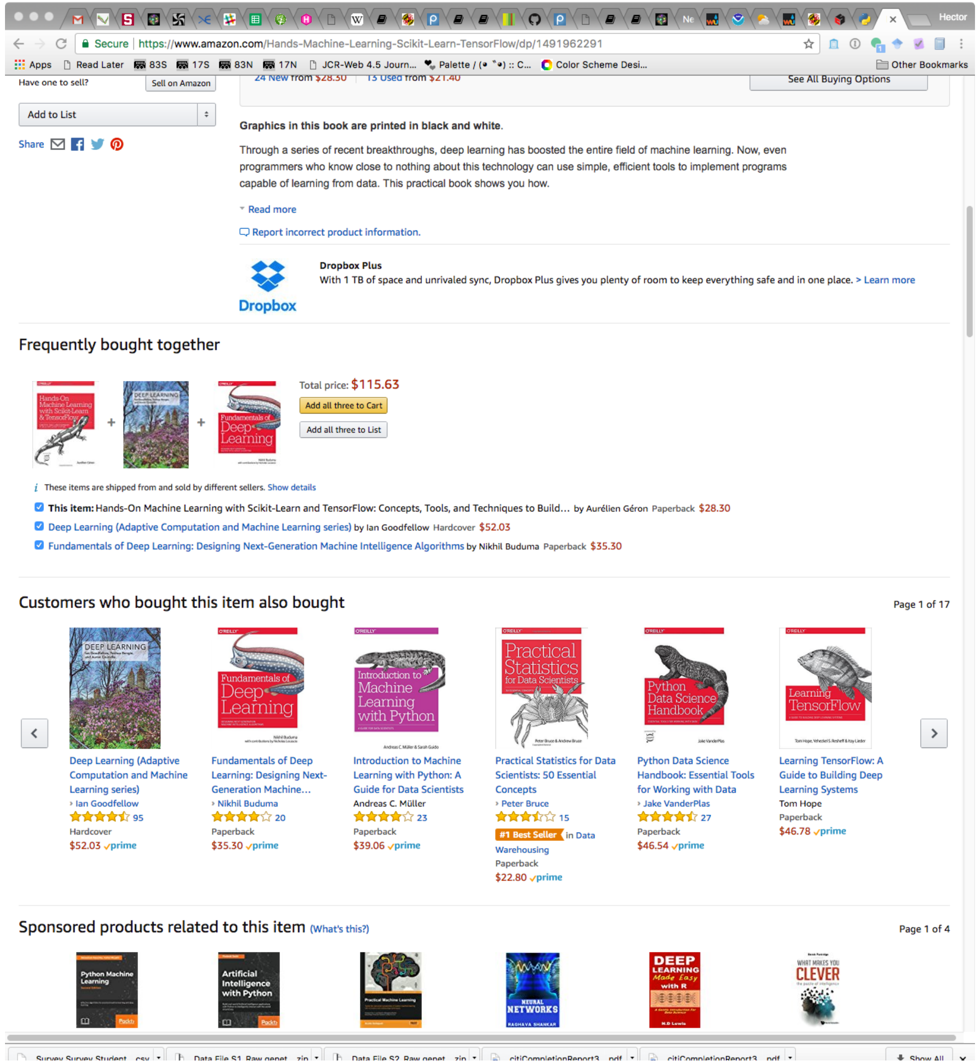
- Items: words
- Itemsets: news articles
- Similar itemsets: news articles usinbg many of the same words



Frequent Itemsets

Online Purchasing

- Items: books
- Itemsets: orders (sets of books)
- Frequent itemsets: sets of books that are purchased together frequently



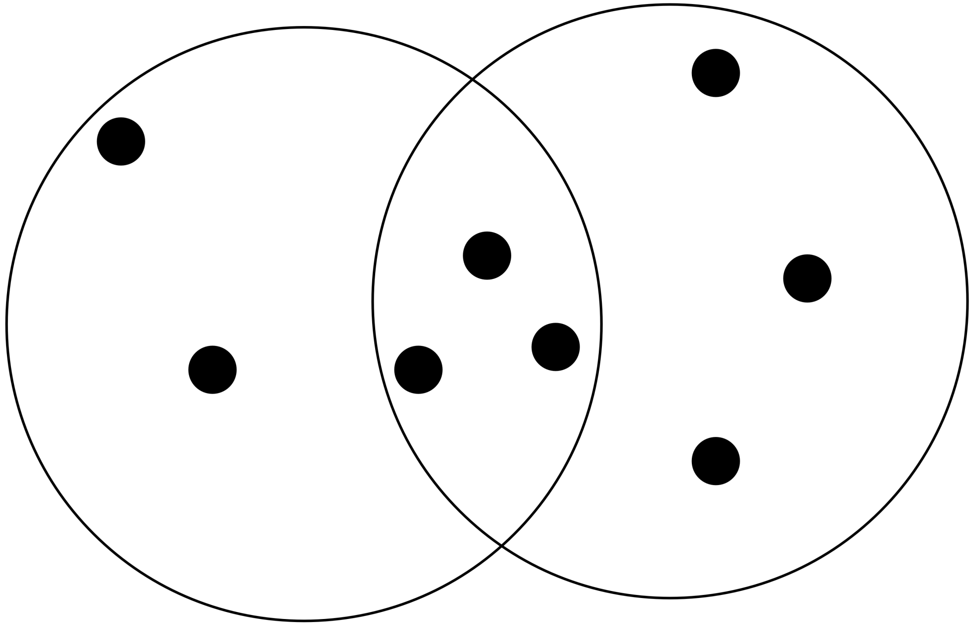
Similar Itemsets

- Describing set similarity (Jaccard Similarity)
- Representing documents as sets (Shingling)
- Similarity-preserving set summaries (Minhash)
- Search for similar itemsets using Locality-Sensitive Hashing (LSH)

Jaccard Similarity

The *Jaccard Similarity* of sets S and T is

$$\frac{S \bigcap T}{S \bigcup T}$$



Exercises

- Compute the Jaccard bag similarity of each pair of sets: $\{1, 1, 1, 2\}$, $\{1, 1, 2, 2, 3\}$, $\{1, 2, 3, 4\}$
- Suppose we have a universal set U of n elements. We chose two subsets S and T, each with m of the n elements. What is the expected value of the JS of S and T?

Documents (Shingles)

- Set all words to lowercase, remove all whitespace and punctuation

```
"Hurricane Irma, they confirmed landfall" ->
"hurricaneirmatheyconfirmedlandfall"
```

Documents (Shingles)

- Set all words to lowercase, remove all whitespace and punctuation

```
"Hurricane Irma, they confirmed landfall" ->
"hurricaneirmatheyconfirmedlandfall"
```

- For some parameter k, represent document as the set of k-long subsequences of document

For k=3

```
{hur,urr,rri,...,eir,irm,rma,...,fir,irm,rme,...}
```

Documents (Shingles)

- Choosing k: choose large enough that probability of any given shingle appearing in any given document is low. Depends on collection.
- Hashing: hash k-shingles instead of using them directly in algorithms that follow
- Using words, effective for similarity (more meaning) but sparser, set of possible shingles is huge

Min-Hash

Clever idea: let's summarize item(sets) (reduce data size!) but make it easy to find similar item(sets).

Min-Hash

Characteristic Matrix

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

Min-Hash

- Permute the rows of the characteristic matrix
- Min-Hash value of set: first non-zero row in corresponding column

Min-Hash

Permuted characteristic Matrix

Element	S_1	S_2	S_3	S_4
b	0	0	1	0
e	0	0	1	0
a	1	0	0	1
d	1	0	1	1
c	0	1	0	1

Min-Hash

Permuted characteristic Matrix

Element	S_1	S_2	S_3	S_4
b	0	0	1	0
e	0	0	1	0
a	1	0	0	1
d	1	0	1	1
c	0	1	0	1

$h(S_1) = a,\ h(S_2) = c$

Min-Hash

Property: $Pr(h(S_i) = h(S_j)) = JS(S_i, S_j)$

Pf: on the board

Min-Hash Signatures

- Choose n permutations of rows, and set $h_i(S_j)$ as the Min-Hash given by permutation i of set j
- Represent set j by the *signature* vector of Min-hashes $[h_1(S_j), \ldots, h_n(S_j)]$
- Collect signature vectors into a *signature matrix*

Min-Hash Signatures in Practice

Instead of row permutations, use hash functions h_i over row indices

Let $SIG(i, c)$ be the ith hash of cth element

Initialize: set $SIG(i, c) = \infty$ for all i and c

Row r:

- Compute $h_i(r)$ for all i
- For each column c:
  - If c has a 0 in row r, do nothing
  - If c has a 1 in row r, then for each $i = 1, \ldots, n$:
    set $SIG(i, c)$ to $\min(SIG(i, c), h_i(r))$

Exercise

Element	S_1	S_2	S_3	S_4
0	0	1	0	1
1	0	1	0	0
2	1	0	0	1
3	0	0	1	0
4	0	0	1	1
5	1	0	0	0

$h_1(x) = 2x + 1 \mod 6$ $h_2(x) = 3x + 2 \mod 6$

$h_3(x) = 5x + 2 \mod 6$

JS and Minhashing

Estimate $JS(S_i, S_j)$ as the proportion of rows (hashes) of the signature matrix that are equal for columns S_i and S_j.

Exercise

Prove that if the JS of two sets is 0, then Min-Hash always gives the right answer.

Locality-Sensitive Hashing

Minhash gives a compressed representation of item(sets) that retains similarity

Locality-Sensitive Hashing

Minhash gives a compressed representation of item(sets) that retains similarity

But to find all pairs of similar item(sets) can still take a lot of time

Locality-Sensitive Hashing

Minhash gives a compressed representation of item(sets) that retains similarity

But to find all pairs of similar item(sets) can still take a lot of time

LSH gives us a way of only comparing likely similar pairs.
Conversely, ignore pairs that are unlikely similar

LSH for Minhash

- Divide signature matrix into b bands, each with r rows
- For each column (itemset) and band, hash it's r entries according to some hash function
- Use same hash function in each of the bands, but use different hash arrays

LSH for Minhash

- Divide signature matrix into b bands, each with r rows
- For each column (itemset) and band, hash it's r entries according to some hash function
- Use same hash function in each of the bands, but use different hash arrays

Itemsets with similar signatures will hash to the same bucket with some likelihood (candidate similar pair)

LSH for Minhash

- Divide signature matrix into b bands, each with r rows
- For each column (itemset) and band, hash it's r entries according to some hash function
- Use same hash function in each of the bands, but use different hash arrays

Itemsets with similar signatures will hash to the same bucket with some likelihood (candidate similar pair)

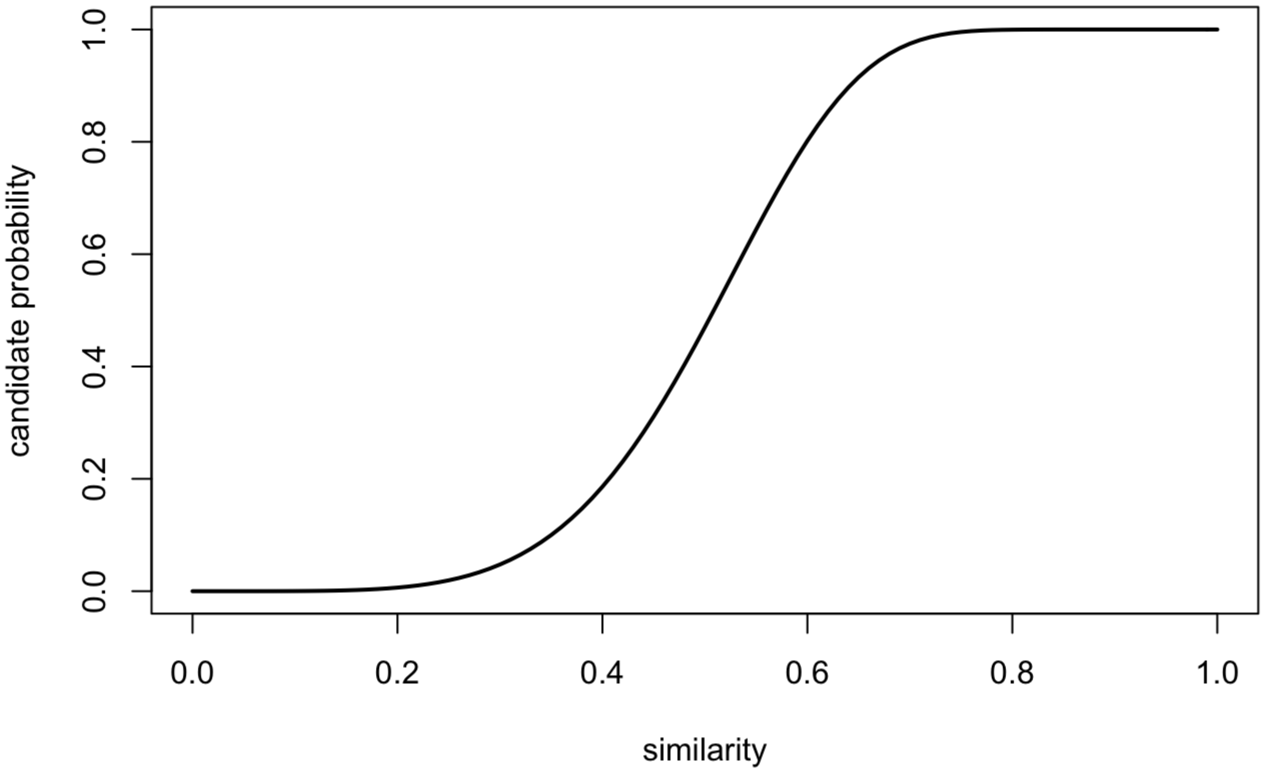
Itemsets without matching signatures will not

Analysis of LSH

Let $JS(S,T) = s$

- Probability signatures agree in all rows of one band: s^r
- Probability do not agree in at least one row of a band: $1 - s^r$
- Probability that signatures do not agree in all rows of any of the bands: $(1 - s^r)^b$
- Probability that signatures agree in all the rows of at least one band (hash to the same bucket at least once): $1 - (1 - s^r)^b$.

Analysis of LSH



Final algorithm for similar document search

Part I: Shingles

- Pick a value of k, construct k-shingles for each document (optionally hashing k-shingles)
- Sort documents by document-shingle pairs by shingle

Final algorithm for similar document search

Part II: Minhash

- Pick a length n for minhash signatures
- Compute minhash signatures for all documents

Final algorithm for similar document search

Part III: LSH

- Choose threshold t for how similar documents have to be to consider as a similar pair
- Choose number of bands b and number of rows r such that $br = n$ and threshold t is approximately $(1/b)^(1/r)$
- Construct candidate pairs using LSH

Final algorithm for similar document search

Part IV: Confirm similar pairs

- For each candidate pair, confirm that their signatures match in at least t fraction of rows
- Optionally, verify similarity in shingled documents

Frequent Itemsets

Find items that occur frequently together in sets

Examples:

- items frequently bought together in the same transaction
- words that appear frequently together in the same document

Market-Basket Model

Items: objects we are modeling Baskets: sets of items (transactions)

Frequent itemsets: items that co-occur frequently in baskets

Frequent Itemsets

Support: define the support of an itemset I as the number of baskets in which itemset I appears

Frequent itemsets: Itemsets I with support at least some support threshold s

Example

(1) {Cat, and, dog, bites}

(2) {Yahoo, news, claims, a, cat, mated, with, a, dog, and, produced, viable, offspring}

(3) {Cat, killer, likely, is, a, big, dog}

(4) {Professional, free, advice, on, dog, training, puppy, training}

(5) {Cat, and, kitten, training, and, behavior}

(6) {Dog, &, Cat, provides, dog, training, in, Eugene, Oregon}

(7) {"Dog, and, cat", is, a, slang, term, used, by, police, officers, for, a, male-female, relationship}

(8) {Shop, for, your, show, dog, grooming, and, pet, supplies}

Association Rules

Rules of the form `I -> j`: if itemset I is in basket, then item j is likely in basket as well

rule confidence: ratio of support of $I \bigcup \{j\}$ to support of I.

rule interest: difference between confidence of rule and fraction of baskets that contain j

Association Rules

Note: once we have itemsets, we can get association rules easily

Association Rules

Note: once we have itemsets, we can get association rules easily

Suppose we find all frequent itemsets over some support threshold

Let itemset J with n items be one of those itemsets, then

1. there are only n candidate Association Rules $J - \{j\} \rightarrow j$
2. Both $J - \{j\}$ and j are also frequent itemsets, so we have already calculated their support
3. We can quickly compute the *confidence* and *interest* of each rule

Exercise

Suppose there are 100 items, numbered 1 to 100, and also 100 baskets, numbered 1 to 100.

Item i is in basket b iff i divides b with no remainder

- Item 1 is in all baskets, item 2 in the even-numbered baskets
- Basket 24 contains items {1,2,3,4,6,8,12,24}

a) If support threshold is 5, which items are frequent? b) Which pairs are frequent?

Itemset monotonicity

If I is a frequent itemset, then every subset of I is a frequent itemset

Why?

The A-priori algorithm

Suppose we are given baskets over n items

First pass

Count the number of occurrences of each item (array of n values)

After first pass

Identify frequent singletons (above support threshold)

The A-priori algorithm

Second pass

Count the number of occurrences of pairs of frequent items

- For each basket:
    - Check which of its items are frequent (first pass)
    - For each pair of items increase occurrence count

After the second pass

Identify frequent pairs (above support threshold)

The A-priori algorithm

Third pass

Count the number of occurrences of frequent pairs + a frequent item

- For each basket:
  - Check which item pairs and singletons that are frequent (first and second pass)
  - For each combination of pair and singleton, increase occurrence count

After the third pass

Identify frequent triples (above support threshold)

The A-priori algorithm

And so on until no more frequent sets are identified

Notes:

- The data structure to store pair counts will be important consideration
- The algorithm has a construct-filter structure: at each pass, *construct* the set of candidate itemsets, *filter* to those that are frequent

Exercise

Apply A-priori algorithm to previous exercise

Handling large datasets

For large datasets storing occurrences of candidate frequent pairs is problematic

PCY algorithm: hash item pairs and keep count in hash bucket

Define candidate frequent pairs as

- i and j anre frequent items
- $\{i, j\}$ hashes to a frequent bucket (with count > threshold)

Handling large datasets

Identify frequent buckets with a bitmap (little memory)

Only count (and verify) candidate pairs as defined above (expected to be much fewer)

Exercise

Consider baskets over items $1, \ldots, 6$

$$\{1,2,3\}\ \{2,3,4\}\ \{3,4,5\}\ \{4,5,6\}$$
$$\{1,3,5\}\ \{2,4,6\}\ \{1,3,4\}\ \{2,4,5\}$$
$$\{3,5,6\}\ \{1,2,4\}\ \{2,3,5\}\ \{3,4,6\}$$

- Compute support for each item and each pair of items
- Using hash function $i \times j \mod 11$ (hash table with 11 buckets), which pairs hash to the same buckets?

Exercise

- Which buckets are frequent?
- Which pairs are counted in the second pass of PCY algorithm?

Summary

Itemset analysis: applications to collaborative filtering, recommendation engines

Finding Similar Itemsets

- Jaccard similarity: measure of set similarity based on common items
- Minhashing with LSH: effective way of finding similar itemsets with efficient data structures for large datasets

Summary

Finding Frequent Itemsets

- Market-basket data: model of item transactions
- Frequent Itemsets: Sets of items appearing frequently in "baskets"
- Association Rules: $I \to j$
- Pair-counting Bottleneck: frequent itemset mining memory space taken mostly in keeping counts of pairs of frequent items
- Monotonicity of frequent itemsets
- A-priori Algorithm
- Hashing for large datasets